On-The-Fly Synchronization Using Wavelet and Wavelet Packet OFDM

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***Abstract*— Wavelet analysis has strong advantages over Fourier analysis, as it allows a time-frequency representation domain, allowing optimal resolution and flexibility. Although OFDM principle has been proposed long time ago, its implementation is quite new and still raises some challenges as frame synchronization. Moreover, wavelet analysis has been proposed only recently for OFDM. While classical synchronization methods use cyclic prefix and Maximum Likehood function or learning sequences, a new dynamic method of synchronization using wavelet OFDM with Haar wavelet is proposed in this paper. Besides, while Fourier OFDM synchronization has been subject to intensive research, synchronization using Wavelet OFDM has not been so deeply studied. Based on the properties of such wavelets, two algorithms are set up for synchronization either with Wavelet or Wavelet Packet OFDM. Simulations show that these techniques provide low-computational, fast and quite robust synchronization methods.**

I. Introduction

OFDM systems are heavily used both in wired and wireless systems. Their performances make them a suitable choice for high data rate and flexible transmission ([21]). Although nowadays the classical implementation still uses Fourier analysis, wavelet and wavelet packet analysis have also been introduced for this modulation scheme ([15], [3], [6], [10]). Wavelets present some decisive advantages, but also some drawbacks such as a greater sensitivity to imperfect sampling, over the classical analysis ([1], [2], [14], [20], [19]). In particular wavelet analysis can facilitate better time-frequency localization which can result in improved signal spectrum use and flexibility.

Whether using Fourier or wavelet approaches, challenges still arise in OFDM systems. One great challenge is frame synchronization. Synchronization issues are well described in [4] and [18], and intensive research has been carried out on the subject ([13], [9], [17], [8], [5], [16], [11], [7], [12]); however almost all of these studies consider Fourier OFDM, and those considering wavelets use Fourier OFDM classical analysis. Synchronization issues are generally overcome using either the Maximum Likehood (ML) function approach ([9], [5], [16], [8], [11]) or by leveraging learning sequences ([17]). However in both cases, some performance bottlenecks arise (as non real-time operation or effective transmission rate decrease).

This paper proposes the use of wavelet analysis to provide an OFDM system that is capable of self-synchronization. Mathematical expressions for the time offsets are analyzed and an innovative synchronization method that uses wavelet analysis is proposed for the two commonly used wavelet implementations (*i.e.* Wavelet and Wavelet Packet OFDM). These novel synchronization methods take advantage of wavelet and wavelet packet OFDM characteristics using the Haar wavelet and do not need learning sequences. They also facilitate the performance of synchronization *before* the transform at the receiver side, or in other words perform synchronization “On The Fly” (*i.e.* while data are incoming). Through leveraging the inherent properties of wavelets to perform the synchronization they provide efficient data rates and computation times.

The remainder of the paper is organized as follows. Section II describes the general principles of Fourier and Wavelet OFDM. The problem of lost synchronization in Fourier OFDM is investigated in Section III. Section IV gives a simple model of the Wavelet and Wavelet Packet OFDM systems and details the “On The Fly” synchronization approach. Section V presents the results obtained in simulation and the algorithm performances are analyzed. Section VI finally concludes the paper.

II. OFDM Principles

The general principle of OFDM systems is to achieve high-speed data throughput through the use of multiple orthogonal carriers for the transmission of the data. There are a number of ways in which an OFDM system can be implemented and for the purposes of the paper we are interested in Fourier ODFM and Wavelet OFDM.

A. Fourier OFDM

Fig. 1 depicts the basic building blocks at the core of a Fourier OFDM system. The incoming data is mapped to the OFDM subcarriers. Each mapped data symbol (*i.e.* frequency domain sequence symbol) corresponds to a single subcarrier. The Fourier OFDM uses the IDFT to convert the mapped data symbols to the time domain. The resulting *OFDM Symbol* is the time-domain waveform representation of the total number of multiplexed sub-carriers. The OFDM symbols are then converted to a serial format for transmission. A cyclic prefix

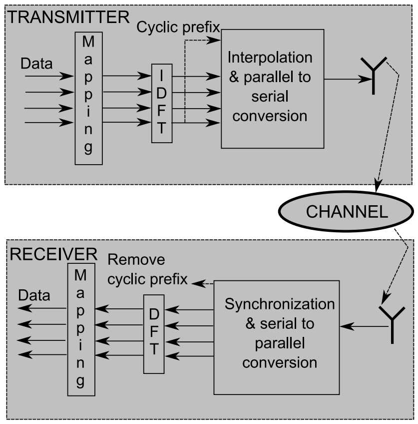


Fig. 1. Block diagram for Fourier OFDM

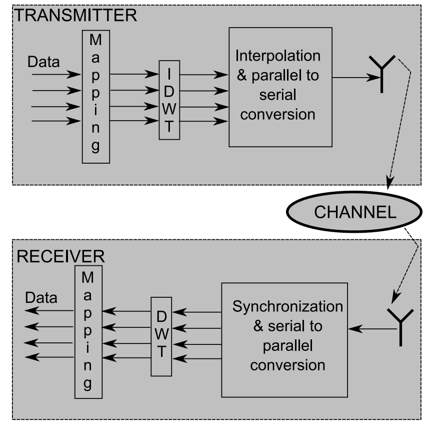
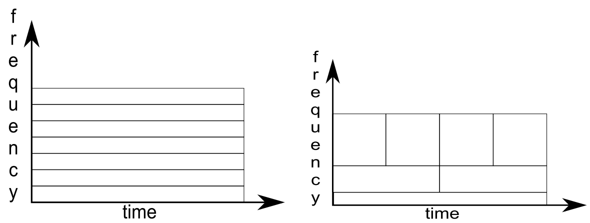


Fig. 3. Block diagram for (Packet) Wavelet OFDM



(a) Fourier OFDM symbol (b) Wavelet OFDM symbol

Fig. 2. Time/Frequency representation

is typically added to the OFDM symbol to avoid Inter-Symbol Interferences (*ISI*). On the receive side the reverse operation takes place. The time/frequency representation of the Fourier OFDM is represented by *Fig. 2(a)*. As stated above each mapped data symbol corresponds to a single subcarrier. In *Fig. 2(a)* each rectangle corresponds to a data symbol and all data symbols are of equal duration in time.

It is obvious here, that due to the nature of the transform used the ML function can only be applied either at the data symbol level (*i.e. after* the direct transform) or on a whole OFDM symbol, (*i.e. before* the transform). Indeed, each part of the OFDM symbol uses all the data sequence, and thus each datum of the OFDM symbol depends on all the data symbols (*i.e.* is the combination of all the frequencies). However the data symbol level ML technique needs to recompute the DFT until the sequence is synchronized, necessitating high computational requirements. The second method, although low computational, leads to a great decrease in the efficient data rate. This last drawback also arises in learning sequence, where a dedicated sequence is sent to synchronize the clocks.

B. Wavelet OFDM

The principle of Wavelet OFDM is almost the same as Fourier OFDM. The main difference is that the IDFT and DFT are replaced by IDWT (Inverse Discrete Wavelet Transform) and DWT respectively. The block diagram is presented *Fig. 3*. One other difference to note, as discussed by Latif and Gohar ([2]), is that the Cyclic Prefix is not helpful for Wavelet OFDM and hence does not appear in *Fig. 3*.

There are two general approaches to implement wavelet transforms. The first, detailed in [1], [2], [15] and [14], is denoted as classical wavelet decomposition and recomposition. The second method, presented in [1], [20], [19], [6] and [10], uses full decomposition and recomposition trees, and is called Wavelet Packet.

1) Classical Wavelet OFDM: In classical wavelet analysis the signal is coded using a recomposition tree as shown in the bottom half of *Fig. 4* for a 16 symbol data sequence. $h[n]$ and $g[n]$ are the low-pass and high-pass filters respectively, defined by the wavelet family of choice. The transmitter first splits the data sequence, filters each of the subsequences to create the resulting OFDM symbol. The OFDM sequences are then processed in a serial fashion for transmission. This approach leads to the time/frequency representation shown in *Fig. 2(b)*, where each block represents a single data symbol. Each of these blocks has the same area, but symbols mapped to higher frequencies have shorter time durations [1]. When the receiver detects a new OFDM symbol it filters it using the decomposition tree given by *Fig. 4*. The obtained sequences are concatenated and the original data sequence is recovered.

2) Wavelet Packet OFDM: In the wavelet packet approach, the signal is coded using a recursive filter-bank such as that depicted in the bottom half of *Fig. 5* for a sequence of length 8. The transmitter takes each data symbol of the time domain sequence, upsamples them and filters them. Then the obtained odd sequences is added to the obtained even sequences, and also upsampled and filtered. The process is repeated until the obtained sequence is equal to the time sequence length. In this case, the time/frequency representation leads to a more complex structure than the previous methods[2]. The receiver part computes the decomposition presented *Fig. 5* to convert the data back in the frequency domain. The received OFDM

[1]typically each symbol lasts one frequency period.

[2]However some particular properties of carrier occupancy can be found. For example the symbol mapping is such that the difference between two consecutive symbols on the same carrier gives the half of the OFDM symbol size. As well for each symbol, the finite sum $\sum_l f_l.tl$ (where f_l and tl are the frequency and time occupancy respectively) gives the same result whatever the symbol, for energy conservation reasons.

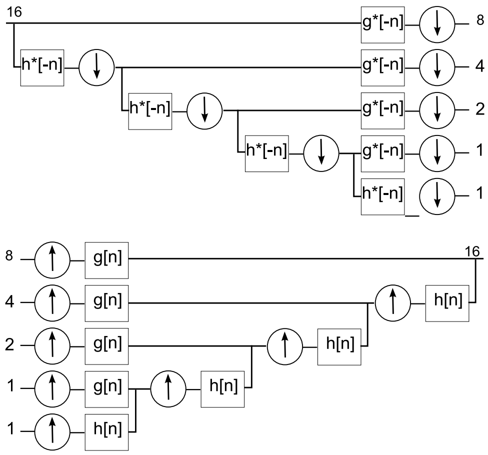


Fig. 4. Decomposition and recomposition filter banks in Wavelet OFDM using 16 data symbol sequence

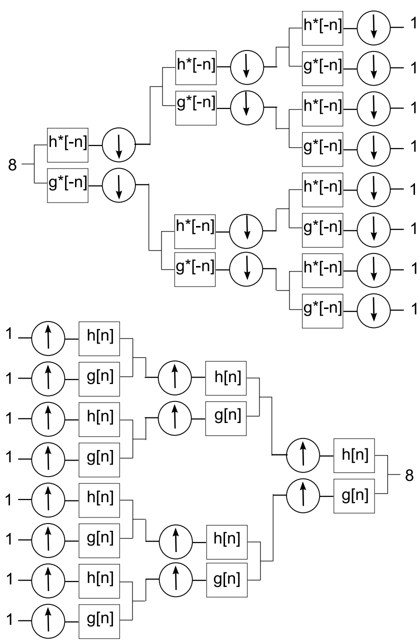


Fig. 5. Decomposition and recomposition filter banks in Wavelet Packet OFDM using 8 data symbol sequence

symbol is filtered, and each filter output is downsampled, and filtered again until 1-symbol sequences are obtained. These sequences are then concatenated to form the original time sequence.

C. Distinctions between Fourier, Wavelet and Wavelet Packet OFDM

As outlined before, the main advantage of Wavelet OFDM over Fourier OFDM is the use of the time-frequency representation offered by wavelets. As shown by *Fig. 2(b)* for example, the data rate is adapted to the carrier frequency, as each symbol datum occupies one period of its assigned sub-carrier. Moreover Wavelet and Wavelet Packet do not need Cyclic Prefix for synchronization, leading to a better symbol rate ([2]). The filter bank approach brings some distinctive advantages for implementation in systems such FPGAs. It is also possible to envisage a dynamic implementation that allows a filter to be added or removed, allowing a varying OFDM symbol length[3]. The classical implementation leads to a similar complexity (in the case of General-Purpose Processors) for all of the methods ([20]). While all of these distinctions are welcome, in terms of this paper however, the main distinction to be made is in the context of its role in synchronization. To see this we first look at the implications of offsets in the Fourier case.

III. Offset effect in Fourier OFDM

Although Fourier OFDM is a very attractive solution for data transmission, it is well known that an offset in the OFDM symbol can prevent the time data sequence from being reconstructed. To understand the impact of an offset, a mathematical modelling is presented. Consider a signal $X[k]$ of length N, transmitted using the architecture depicted in *Fig. 1*, the corresponding transmitted signal $x[n]$ is given by (1). Thus the decoded received signal (synchronized) is given by (2). The term in brackets is non zero only for $p-k=0 \pmod N$ and in this case equals N. Considering a single OFDM symbol (*i.e.* $p,k<N$) we thus obtain the original sequence. However, if the signal is not synchronized an offset of m samples is introduced (just before the DFT block), and this expression is no longer valid. Two cases have to be considered: a right shift that leads to (3) and a left shift that leads to (4). We can see that the first term corresponds to a pure phase shift, and the second term to both a modulation and a phase shift. Hence, the signal is lost.

$$x[n] = IDFT(X[k]) = \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn} \tag{1}$$

$$\begin{aligned} X_r[p] &= DFT(x[n]) \\ &= \sum_{k=0}^{N-1} X[k] \frac{1}{N} \left(\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-p)n} \right) = X[p] \end{aligned} \tag{2}$$

$$\begin{aligned} X_r[p] &= DFT(x[n+m]) = X[p] e^{j\frac{2\pi}{M}pm} \\ &- \sum_{k=0}^{N-1} X[k] e^{j\frac{\pi}{N}(p(m+1)+k(m-1))} \left(\frac{sin\left(\frac{\pi}{N}(k-p)m\right)}{sin\left(\frac{\pi}{N}(k-p)\right)} \right) \end{aligned} \tag{3}$$

$$\begin{aligned} X_r[p] &= DFT(x[n+m]) = X[p] e^{j\frac{2\pi}{M}pm} \\ &- \sum_{k=0}^{N-1} X[k] \frac{e^{j\frac{\pi}{N}(2N-m-1)k}}{e^{j\frac{\pi}{N}(2N+m-1)p}} \left(\frac{sin\left(\frac{\pi}{N}(k-p)m\right)}{sin\left(\frac{\pi}{N}(k-p)\right)} \right) \end{aligned} \tag{4}$$

The previous equations stand for the case of an integer frequency offset. However the sent signal is generally interpolated, and thus the offset can also be fractional. Let the incoming symbol $x_r[l]$ as (5), where δ is the fractional part of the offset and $f(\delta)$ the interpolation function between two symbols $x[n]$ and $x[n+1]$. Thus, the reconstruction of the signal is given by (6).

[3]Wavelet Packet can also be implemented using a recursive approach, allowing cost-effective implementation and better computation cost.

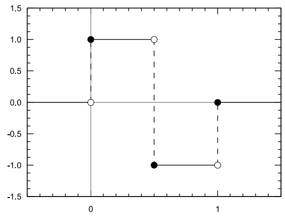


Fig. 6. Haar wavelet

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Fig. 7. Normalized Haar decomposition/recomposition matrix

$$\left[\begin{array}{cccc|cc|c|c} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0.5 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0.5 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & -0.5 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & -0.5 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0.5 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0.5 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -0.5 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & -0.5 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{array}\right]$$

Fig. 8. 8 data symbol coding matrix for Wavelet OFDM (with section outline)

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad G = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \end{bmatrix}$$

Fig. 9. Low and high-pass basic filter matrices

$$x_r[l] = (1 - f(\delta))x[n] + f(\delta)x[n+1] \tag{5}$$

$$\begin{aligned} &DFT(x_r[l]) \\ &\quad = (1 - f(\delta))DFT(x[n]) + f(\delta)DFT(x[n+1]) \end{aligned} \tag{6}$$

IV. On-The-Fly Synchronization

Two methods of synchronization are now proposed for Wavelet OFDM and for Wavelet Packet OFDM approach respectively. These proposals provide new insights and techniques, as the large majority of Wavelet and Wavelet Packet OFDM synchronization techniques rely on methods used in Fourier OFDM.

A. Wavelet and Wavelet Packet OFDM Modelling

As the two wavelet OFDM systems are different, a model is defined for each of them. However a common modelling strategy is adopted. The purpose of the models is to express the transforms using polyphase matrices so that properties of interest can be identified, and exploited for synchronization purposes. We can note that these matrices are orthogonal (*i.e.* ${}^tA = A^{-1}$), which is a logical consequence of OFDM. The normalized Haar wavelet is used here. The Haar mother wavelet is shown *Fig. 6*, and has the same decomposition and recomposition matrix given by *Fig. 7*. This is the simplest and the first known wavelet (proposed in 1909 by Alfred Haar), but has very interesting properties, such as orthogonality and equal matrices for both decomposition or recomposition.

1) Wavelet OFDM: The filter bank approach proposed *Fig. 4* can be represented as a polyphase N-by-N matrix which can be decomposed into particular sections (*c.f. Fig. 8*), reflecting the time/frequency representation presented in *Fig. 2(b)*. Indeed, the first half of the columns corresponds to the highest frequency, the third quarter of the columns to the second highest frequency, and so on. More generally, each half of the remainder of the columns has $N - 2^j$ zeros and $\pm(\sqrt{2})^{-j}$ on the diagonal, with $j = 1, \ldots, log_2(N)$. The last column contains N times $(\sqrt{2})^{-log_2(N)}$, and correspond to the normalized average value of the signal. An example of a 8 symbol frame coding matrix is given *Fig. 8*.

2) Wavelet Packet OFDM: Here we can also express the Wavelet Packet transform as a polyphase matrix. Basically the matrix is constructed recursively, starting from the 1D low and high-pass filters given by *Fig. 9*. These operation can be represented as *Fig. 10*, where A_{k+1} is the resulting square matrix and A_k the square matrix of the previous stage. N_k is the dimension of the A_k matrix, with $N_k = 2^k$, $k = 1, \ldots, log_2(N) - 1$ (A_1 is equals to the Haar decomposition/recomposition matrix given by *Fig. 7*).

In addition of being an orthogonal matrix, another remarkable property (7) arises here, where A is the transform matrix, ^{t} the transpose operator and * the conjugate operator. This matrix is consequently also Hermitian, and so defines an automorphism of order 2. An example of such a matrix for 8 symbol frame coding is given *Fig. 11* [4].

$${}^tA = A^* = A^{-1} \tag{7}$$

B. Wavelet OFDM

The previous exposed system model allows to express the reverse process by the transpose of the matrix described in *Section 4.A.1*. This matrix contains a certain number of zeros on each line. The number of zeros gives the relative carrier frequency and the place of non-zero values the temporal place of the corresponding coded symbol. Thus applying the matrix

[4] We can illustrate here the change in the carrier occupancy of a particular coded symbol. Indeed, considering the 8^{th} line, which correspond to the 8^{th} coded symbol, if one considers the first four values, the apparent frequency is 2 (sequence $+\ -\ -\ +$), but considering the four values in the middle yields an apparent frequency of 4 (sequence $-\ +\ -\ +$)

$$A_{k+1} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_k(1,:) & A_k(1,:) \\ A_k(1,:) & -A_k(1,:) \\ A_k(2,:) & A_k(2,:) \\ A_k(2,:) & -A_k(2,:) \\ \vdots & \vdots \\ A_k(N_k,:) & A_k(N_k,:) \\ A_k(N_k,:) & -A_k(N_k,:) \end{bmatrix}$$

Fig. 10. Matrix recursive expression for Wavelet Packet OFDM

$$\frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

Fig. 11. 8 data symbol coding matrix for Wavelet Packet OFDM

to a shifted sequence would lead to a wrong reconstruction. However, the properties of the matrix can be exploited to correct this shift. Basically, as each line contains the frequency *and* the time information of the signal, an error detection would give an estimation of the offset. Considering only integer shift, an error in the first half of the decoded symbol means that there is an odd shift. An error in the third quarter means that there is an offset between 1 and 2^2-1, and so on. Taking as example *Fig.8*, an error in the first section infers an odd offset, in the second section an offset between 1 and 3, and in the two last sections an offset between 1 and 7. Thus by detecting errors and analyzing their place, the offset can be estimated and thus corrected.
The algorithm described in *Fig. 12* performs these operation. An error at the k^{th} stage would lead to a shift of the analyzed sequence by 2^{k-1}, $k=1,\dots,log_2(N)-1$ with N the length of the sequence. The last step analyzes the last two lines together to determine an offset of $\frac{N}{2}$.

In the case of non-integer offsets, the resulting decoded symbol can be expressed in almost the same way than in the Fourier OFDM as (8), and is more exactly the balanced sum of the filter outputs. The algorithm modifications to take in account the non-integer offsets are very slight. The shift of the sequence is one sample period for the first stage, and then for all the other stages the shift is given as $2^{k-1}N_{interp}$, where N_{interp} is the number of interpolation points.

$$\begin{aligned} &DWT(x_r[l]) \\ &\quad = (1-f(\delta))DWT(x[n]) + f(\delta)DWT(x[n+1]) \end{aligned} \tag{8}$$

C. Wavelet Packet OFDM

In Wavelet Packet OFDM, a particular relationship between each coefficient and the transmitted sequence can be outlined (*e.g.* (9), where $a[n]$ are the original symbols and $b[n]$ the received coded symbols, $n=0,\dots,N-1$, with N the frame size). From this observation it is so possible to construct a Maximum Likehood (ML) function that allows synchronization using few symbols, and *before* the DWT. For example the ML function can be taken as the squared difference sum between the received symbol and the synchronization data, yielding (10) for a synchronization using the first two data (thus $a[0]$ and $a[1]$ are fixed and known), with K an arbitrary parameter representing the detection sensivity. The simple relationship thus allows a fast but efficient synchronization.

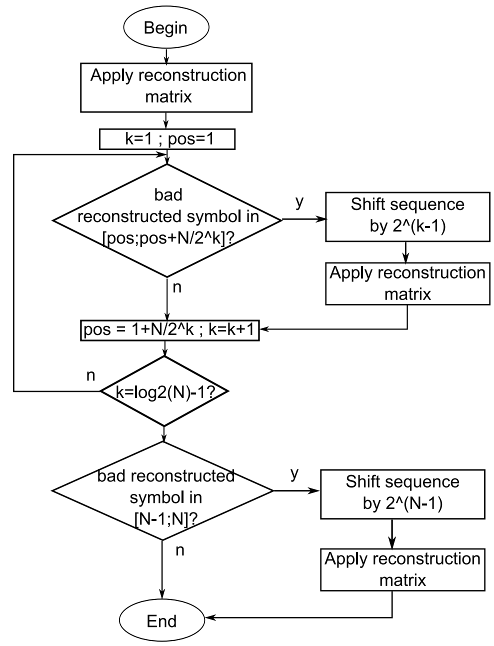


Fig. 12. Wavelet OFDM Auto-Synchronization algorithm

We can note here that the robustness and efficient data rate form a trade-off. As well, this ML algorithm automatically takes into account non-integer offsets.

$$\begin{aligned} \mathbf{a[0]} &= \tfrac{1}{\sqrt{N}} \textstyle\sum_{n=0}^{N-1} b[n] \\ \mathbf{a[1]} &= \tfrac{1}{\sqrt{N}} \left[\textstyle\sum_{n=0}^{\frac{N-1}{2}} b[n] - \sum_{n=\frac{N-1}{2}+1}^{N-1} b[n] \right] \\ \mathbf{a[2]} &= \tfrac{1}{\sqrt{N}} \Big[\textstyle\sum_{n=0}^{\frac{N-1}{4}} b[n] - \sum_{n=\frac{N-1}{4}+1}^{\frac{N-1}{2}} b[n] \\ &\quad + \textstyle\sum_{n=\frac{N-1}{2}+1}^{3\frac{N-1}{4}} b[n] - \sum_{n=3\frac{N-1}{4}+1}^{N-1} b[n] \Big] \\ \mathbf{a}\left[\tfrac{\mathbf{N}}{\mathbf{2}}\right] &= \tfrac{1}{\sqrt{N}} \left[\textstyle\sum_{n=0}^{\frac{N-1}{2}} b[2n] - b[2n+1] \right] \end{aligned} \tag{9}$$

$$\begin{aligned} &\left(a[0] - \tfrac{1}{\sqrt{N}} \textstyle\sum_{n=0}^{N-1} b[n] \right)^2 \\ &+ \left[a[1] - \tfrac{1}{\sqrt{N}} \left(\textstyle\sum_{n=0}^{\frac{N-1}{2}} b[n] - \sum_{n=\frac{N-1}{2}+1}^{N-1} b[n] \right) \right]^2 < K \end{aligned} \tag{10}$$

V. SIMULATION

In this Section the previous proposed methods of synchronization are tested and analyzed. After defining a simulation strategy, obtained results are analyzed and the performances compared with classical OFDM results (*Fig. 13*). In order to compare the performance with almost the same complexity level, the classical OFDM synchronization is performed using a simple ML function (squared difference sum) on 4 data. However, one can note that it has been observed that the classical Fourier OFDM synchronization takes much more time than the Wavelet OFDM synchronization.

A. Simulation Strategy

The principles of the simulation are to repeat the proposed synchronization methods few (ten) times for a given offset on

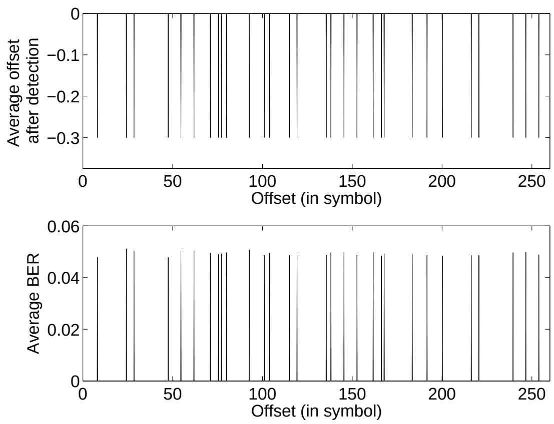


Fig. 13. Fourier OFDM Auto-Synchronization simulation results

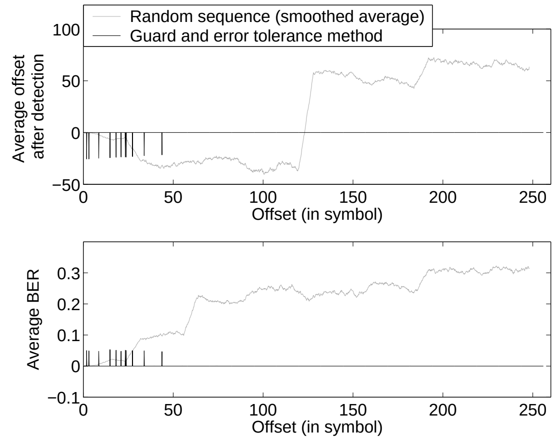


Fig. 14. Wavelet OFDM Auto-Synchronization simulation results

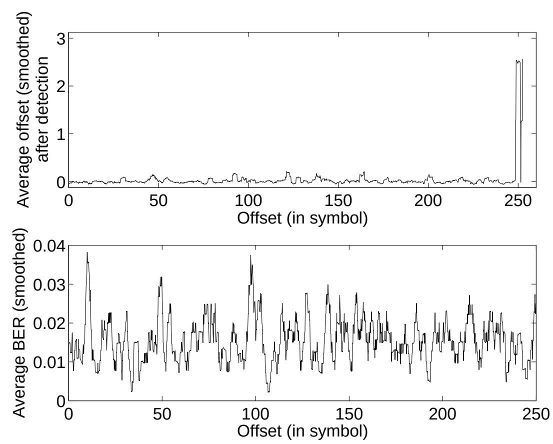


Fig. 15. Wavelet Packet OFDM Synchronization simulation results

randomly generated data. Then the average BER and resulting offset are calculated for each offset. The frame size is taken as 256 symbols and the interpolation is classically made using upsampling and FIR filtering on 8 points. The Signal to Noise Ratio is taken as 20 dB over an Addititive White Gaussian Noise (AWGN) channel. The symbols are encoded using QAM-16 modulation. In the case of the Wavelet OFDM, a symbol is declared invalid if its distance from the nearest QAM constellation point is greater than a threshold. This threshold is tightly coupled to the SNR, and is fixed to $0.7\sqrt{2}$ in our case (empirically determined).

The simulation for Wavelet Packet OFDM uses a ML function taken as the least squared error sum over a frame duration (see example (10)). This ML function applies to the first three symbols and to the mid-symbol of the sequence, which are fixed (*c.f.* (9)). It is also ensured that this particular sequence does not appear elsewhere in the transmitted sequences, which are also chosen randomly.

The robustness of the methods are compared by performing several simulations for different SNR. For each SNR value, ten randomly chosen offsets are applied to the system. As well, transmission simulations over a simple fading channel are performed. The model of the fading channel is taken as the convolution of each symbol with $f = 1 + \sigma^2 \left(N(0,1) + jN(0,1)\right)$, with N the gaussian distribution.

B. Results & Performance Discussion

Results for the Wavelet OFDM are presented in *Figs. 14*. As the sequences are randomly chosen for the first set of results, there can be a coupling between two values of two consecutive sequences. Thus, an error is not always detected. Particularly, the probability of false offset detection increases with the offset[5]. The second set of plots for the Wavelet OFDM synchronization method represents the effectiveness of the method if guard intervals are added between OFDM symbols. The guard interval length is set to $\frac{1}{8}$ of the frame length. Also, this set allows few invalid symbols, in order to comply with the invalid symbols introduced by the noise (*i.e.* not by an offset). This technique is thus much more robust facing the noise, but also increases the effectiveness of the method in normal operation condition, especially for high offsets.

Results for Wavelet Packet OFDM are presented *Fig. 15*. The offset is rather well detected with the least squared error sum. The synchronization is well performed in most of the cases[6].

The robustness of the methods are tested for several Signal to Noise Ratios in a AWGN channel (modeling a composition of transmission channel models). Results are given in *Fig. 16*. The threshold for this algorithm is given by the empirical function $0.5\left(1 + e^{-\frac{SNR^2}{1700}}\right)$, which has however its origin in a gaussian distribution. The results show that the synchronization method has a minimum admissible Signal to Noise Ratio of approximatively 15 dB (17 dB for Fourier OFDM). Compared to Fourier OFDM, one can observe that the offset is not so well detected. However one can keep in mind that

[5]As the size of the matrix corresponding to a particular stage of error detection decreases with the stage number (detection of higher offsets), the number of possibilities of detection also decreases, and the probability of false offset detection increases. This is well illustrated in *Fig. 14* by the steps in the false detection rate (these steps occur typically for offsets equal to 2^k).

[6]We can note that when the offset is close to the frame length, the algorithm can lock to the next frame, which explains the high offset value at the end

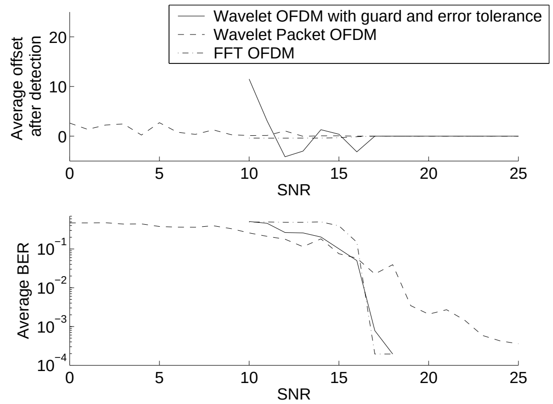


Fig. 16. Wavelet and Wavelet Packet OFDM robustness over an AWGN

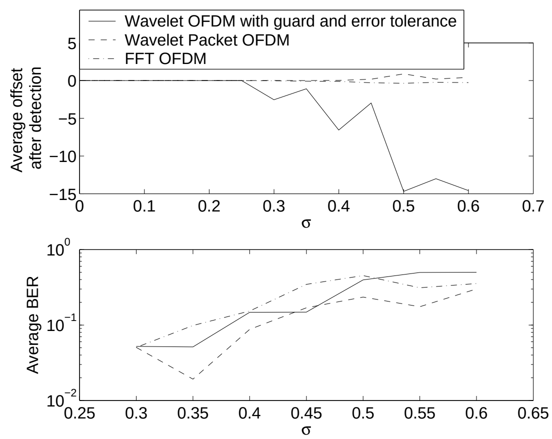


Fig. 17. Wavelet and Wavelet Packet OFDM robustness over a fading channel

the Fourier OFDM algorithm took much longer time. As well, the Fourier OFDM sensitivity to offset makes the BER of each method comparable.

In the same manner, transmission over a fading channel has also been simulated as previously described, with a channel modelling $f = 1 + \sigma^2 \left(N(0,1) + jN(0,1) \right)$. The results are presented in *Fig. 17*. In this case the threshold is fixed to $0.3 \left(1 + e^{2\sigma^2} \right)$. These results shows that the Wavelet and Wavelet Packet OFDM synchronization methods perform in a comparable manner than the Fourier OFDM. As explained before, while the detected offset excursion is greater (especially for Wavelet OFDM), the impact on the BER is almost the same, as Wavelet OFDM schemes are much more robust to small phase shifts compared to Fourier OFDM.

VI. Conclusion & Further Work

Wavelet and Wavelet Packet OFDM provide a very interesting and powerful alternative to Fourier OFDM, the strength of the wavelets being the better time/frequency localization and distribution in this domain. However this characteristic also raises the problem of synchronization, which is already an issue for classical Fourier OFDM, but has greater impact in wavelet domain. In this paper we provide two brand new methods that address this issue, using Haar wavelets. Particularly, the proposed methods are fast and computationally efficient. Their robustness is not as good as advanced methods (which however necessitate huge computation), but the methods operate quite well in classical conditions. Performances have been illustrated in simulation. The synchronization issue is well resolved, and the robustness is acceptable in classical environment but is thought to be possibly enhanced. As well, it is envisaged to test these methods in a real environment, using a reconfigurable software radio platform.

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