ARTICLE IN PRESS

Available online at www.sciencedirect.com

SCIENCE DIRECT®

Applied Mathematics and Computation xxx (2006) xxx–xxx

APPLIED MATHEMATICS AND COMPUTATION

www.elsevier.com/locate/amc

Bicriteria shortest path in networks of queues

Amir Azaron

Cork Constraint Computation Centre, Department of Computer Science, University College Cork, Cork, Ireland

Abstract

In this paper, I propose a new methodology to find the bicriteria shortest path from the source to the sink node of a network of queues under the steady-state condition. I assume the number of servers in each service station settled in a node of the network is either one or infinity. Moreover, the arc lengths are mutually independent random variables. First, I propose a method to transform each node, containing a service station, into a proper stochastic arc corresponding to the waiting time in the particular service station. Then, the stochastic network is transformed into a bicriteria network by computing the mean and the variance of the waiting time in each service station and augmenting those to the transformed arc. Finally, by defining a proper utility function, dynamic programming is used to obtain the bicriteria shortest path. The time complexity of the proposed algorithm is O(b), considering b as the number of service stations. The proposed method is suitable to find the shortest rout from the origin to the destination in stochastic routing problems. Moreover, this method is useful to approximate the mean and the variance of project completion time in dynamic PERT networks.

Keywords: Queueing; Dynamic programming; Stochastic routing problems; Complexity

1. Introduction

Due to the vast applications of network of queues, it is one of the most important issues in queueing theory. The complexity of analyzing this subject is another reason that some researchers are still investigating it from the various points of view.

Many problems in the fields of transportation networks, production systems, computer networks and distributed processing systems are expressed in the framework of networks of queues. For example, consider the problem of routing a ship in the ocean. In this case, nodes of the network of queues indicate the geographical regions where the ship passes through and the ship should wait in each region (service station) for anchoring and fuelling. The transport times from each region to the adjacent regions are assumed to be mutually independent random variables with general distributions. The length of each path in the network is equal to the sum of the lengths of the arcs and the nodes of this path. The length of each node is equal to the waiting time for anchoring and fuelling in the particular region (waiting time in queue plus the processing time), and the arc lengths are equal to the transport times between the service stations. In each region, the number of servers is

E-mail address: a.azaron@4c.ucc.ie

0096-3003/$ - see front matter
doi:10.1016/j.amc.2006.04.004

either one or infinity. In practice, a queueing system with infinite servers indicates that there is ample capacity so that no ship ever has to wait. Finally, we can obtain the shortest path length from the origin to the destination in this ship routing problem, considering the mean length and also its variance, as two important criteria in routing problems, using the proposed method.

Azaron and Kianfar [1] proposed an exponential algorithm to find the dynamic shortest path in the ship routing problem, but the waiting time in each region for anchoring and fuelling, which is an important factor to compute the shortest path length, has not been considered in this paper. The proposed algorithm in this paper can overcome this drawback.

In the proposed methodology, if a service station exists in any node of the network, then a proper stochastic arc corresponding to the waiting time in the particular service station is augmented to the network. Then, the length of any arc in the transformed stochastic network is replaced by two deterministic criteria corresponding to the mean and the variance of the arc length. Finally, the best bicriteria path is found by using dynamic programming, considering a utility function, which includes two indicated criteria.

There are several papers, which obtain a specific path as the shortest path in stochastic networks, considering different criteria like minimum expected path length, minimum of the variance of the path length or the maximum probability that the shortest path length becomes smaller than a specific value. Lee and Polat [9] presented a method for solving the bicriteria network flows problems. The network flows include a wide range of problems like transportation, maximal flow and shortest path. The efficient extreme points of the bicriteria network are found by a combination of the bicriteria linear programming with the out-of-kilter method. Since the decision variables of the shortest path problem are zero-one, this algorithm does not have enough efficiency for solving the shortest path problem. The main approach for solving the multicriteria shortest path problems is to obtain the efficient paths of the network. For example, Martins [10] found the set of efficient paths for the bicriteria shortest path problem by using dynamic programming. Current and Min [6] provided a taxonomy and annotation for the multicriteria networks. Wijerante et al. [11] presented a method for finding the set of non-dominated paths from the source to the sink node. They assumed that each arc includes several criteria and some of them might be stochastic. They replaced each stochastic criterion with two deterministic criteria corresponding to the mean and the variance of the stochastic criterion. Then they found the set of non-dominated paths by a multicriteria algorithm. Carraway et al. [5] applied the method of generalized dynamic programming for finding the shortest path of a bicriteria network, in which one criterion is corresponding to the shortest path and the other criterion is corresponding to the most reliable path of the network. They proved that DP's monotonicity assumption is violated in this situation. Therefore, they used generalized DP, which avoids the potential pitfalls created by this absence of monotonicity, thereby guaranteeing optimality.

Although there are several papers corresponding to the multicriteria networks and also the multicriteria shortest path problems, but there is no paper about finding the multicriteria shortest path in networks of queues and its applications in different fields like stochastic routing problems and also stochastic dynamic scheduling problems.

2. Shortest path analysis in networks of queues

In this section, first I propose a method for transforming an acyclic network of queues (called original network) into an equivalent stochastic network, by replacing each node containing a queueing system with a stochastic arc. The length of this arc is equal to the waiting time in the queueing system located in the corresponding node of the original network. Then, the stochastic network is transformed into a bicriteria network by computing the mean and the variance of the waiting time in each queueing system and augmenting those to the new arc.

Networks of queues that I consider in this paper include the major properties of Jackson networks including these two properties:

1. The service times are independent random variables with general distributions, and there is only one or infinite servers in each service station. Therefore, we encounter the following service stations: $M/M/1$, $M/M/\infty$, $M/G/\infty$, $M/G/1$ and $G/M/1$.
2. The arc lengths between the service stations are independent random variables with general distributions.

Let's explain how to replace node k of the network of queues, which contains a queueing system, with a stochastic arc. Assume that $b_1, b_2, \ldots, b_n$ are the incoming arcs to this node and $d_1, d_2, \ldots, d_m$ are the outgoing arcs from it. Then, I substitute this node by arc (k', k''), whose length is equal to the waiting time in system for the particular queueing system. Furthermore, all arcs b_i for $i = 1, \ldots, n$ end up with k' while all arcs d_j for $j = 1, \ldots, m$ start from node k''. Therefore, the network of queues is transformed into a stochastic network (see Azaron and Modarres [2] for more details).

Let V and A represent the sets of nodes and the arcs of the transformed stochastic network $G = (V, A)$, respectively. Since the length of each arc in the transformed stochastic network is equal to the waiting time of the queueing system located in the corresponding node of the original network, we need to derive the waiting time distributions of different classes of queuing systems, which will be done in the next sub-sections.

2.1. Networks of queues with M/M/1, M/M/∞ and M/G/∞ queueing systems

Let $w_1(t)$, $w_2(t)$ and $w_3(t)$ represent the density functions of the waiting time in the system in $M/M/1$, $M/M/\infty$ and $M/G/\infty$ service stations, respectively. It is well known that $w_1(t)$ is exponential with parameter $(\mu - \lambda)$, if λ represents the arrival rate to the service station and μ represents the service rate of $M/M/1$. The mean and the variance of the waiting time in this queueing system are $(1/(\mu - \lambda))$ and $(1/(\mu - \lambda))^2$, respectively.

$W_2(t)$ has exponential distribution with parameter μ, if μ represents the service rate of $M/M/\infty$, because the waiting time in system would be equal to the service time with exponential distribution. The mean and the variance of the waiting time in this queueing system are $(1/\mu)$ and $(1/\mu)^2$, respectively.

It is also clear that $w_3(t) = B'(t)$, if $B(t)$ represents the distribution function of service time in $M/G/\infty$, because there is no queue in $M/G/\infty$. We can easily compute the moments of the waiting time in this queueing system, taking into account $B(t)$. It should be noted that the steady-state density function of time between two successive departures from an $M/G/\infty$ queueing system is exponential (see Gross and Harris [7] for more details). Therefore, the departure process from the $M/G/\infty$ queueing system is a Poisson process. The rate of this process is equal to the arrival rate to this queueing system.

2.2. Networks of queues with M/G/1 queueing systems

We cannot easily compute the distribution function of the waiting time in this queueing system like the previous ones, but we can compute the Laplace–Stieltjes transform of the distribution function of the waiting time in system as follows (see Gross and Harris [7] for more details):

$$
\begin{aligned}
W^*(s) &= (1-\rho)sB^*(s)/(s - \lambda(1 - B^*(s))), \\
W^*(s) &= \int_0^\infty e^{-st}\,dW(t),
\end{aligned}
\tag{1}
$$

where λ, ρ and $B^*(s)$ represent the rate of arrival process to $M/G/1$, the utilization factor of $M/G/1$ and the Laplace–Stieltjes transform of the service time distribution function, respectively. Now, we can compute the mean and the variance of the waiting time in system from the following equations and transfer these two criteria to the augmented arc of the network:

$$
\begin{aligned}
\mu^* &= -d/ds W^*(s)|_{s=0}, \\
\sigma^{*2} &= d^2/ds^2 W^*(s)|_{s=0} - \mu^{*2}.
\end{aligned}
\tag{2}
$$

2.3. Networks of queues with G/M/1 queueing systems

This situation arises when there is a service station including one server with exponential service time along the path after passing any $M/G/1$ queueing system. If the arrival to each queueing system follows a Poisson process, then all queuing systems are Morkovian. In this case, let r_{ji} represent the probability of movement of a job from node j to node i and λ_i be the rate of arrival Poisson process to the service station settled in node i. Then,

$$\lambda_i = \sum_{j \in V} r_{ji} \lambda_j \quad i \in V. \tag{3}$$

However, if there is even one $M/G/1$ queueing system in the network, the subsequent queueing systems connected to this one no longer face Poisson arrival processes.

It is assumed that the departure process from an $M/G/1$ queueing system settled in node $u \in V$ has independent increment. This assumption is reasonable, because the network is acyclic. Therefore, we can approximate the departure process from node u as a non-homogeneous Poisson process with the intensity function equal to the hazard rate function of the time between two successive departures from this queueing system. Let $B(t)$ and ρ represent the distribution function of service time and the utilization factor of the $M/G/1$ queueing system settled in node u. Then, $C(t)$ or the distribution function of time between two successive departures from this queueing system is given by (see Gross and Harris [7] for details):

$$C(t) = \rho B(t) + (1-\rho) \int_0^t B(t-u) \lambda e^{-\lambda u} \, du. \tag{4}$$

The rate of departure process from the queueing system settled in node u or $\lambda_u(t)$ is equal to the hazard rate function of the time between two successive departures from this queueing system. Therefore, $\lambda_u(t)$ is given by $\lambda_u(t) = C'(t)/(1 - C(t))$.

Consequently, $N(t)$ or the departure process from the queueing system settled in node u would be a non-homogeneous Poisson process with the intensity function $\lambda_u(t)$ and the following distribution:

$$P[N(t) = m] = e^{-\int_0^t \lambda_u(s)\, ds} \left[\int_0^t \lambda_u(s)\, ds \right]^m \bigg/ m! \quad m \geqslant 0. \tag{5}$$

Let $N_1(t)$ represent the arrival process from the queueing system settled in node u to the queueing system settled in node $v \in V$. Thus, $N_1(t)$ would be a non-homogeneous Poisson process with the intensity function equal to $r_{uv}\lambda_u(t)$, or its distribution is

$$P[N_1(t) = n] = e^{-\int_0^t r_{uv}\lambda_u(s)\, ds} \left[\int_0^t r_{uv}\lambda_u(s)\, ds \right]^n \bigg/ n! \quad n \geqslant 0. \tag{6}$$

Then, it is proved that the rate of arrival process to each $G/M/1$ queueing system, settled in node $k \in V$, is given by (see Azaron and Modarres [2] for the details of proof):

$$\lambda_k(t) = \sum_{j \in V} r_{jk} \lambda_j(t) \quad k \in V. \tag{7}$$

The closed form (7) is quite similar to (3). After computing $\lambda_k(t)$, we can compute the distribution function of time between two successive arrivals to node k or $A(t)$ from the following equation:

$$A(t) = 1 - e^{-\int_0^t \lambda_k(u)\, du}. \tag{8}$$

Assuming μ as the service rate of the $G/M/1$ queueing system, we can compute $0 < x_0 < 1$, or the unique root of Eq. (9), by using a numerical method like the Newton–Raphson method.

$$Z = \int_0^\infty e^{-\mu t(1-z)} \, dA(t) \quad 0 < z < 1. \tag{9}$$

After computing x_0, we can compute the density function of sojourn time in the $G/M/1$ queueing system, by (see Gross and Harris [7] for details):

$$w(t) = \mu(1-x_0) e^{-\mu(1-x_0)t} \quad t > 0. \tag{10}$$

Clearly, $w(t)$ has exponential distribution with parameter $\mu(1-x_0)$, and we can replace the queueing system settled in this node with an exponential arc with parameter $\mu(1-x_0)$. Therefore, the mean and the variance of the waiting time in this queueing system would be $(1/\mu(1-x_0))$ and $(1/\mu(1-x_0))^2$, respectively. Now, we can transfer these two criteria to the augmented arc of the network and finally transform the network of queues into a bicriteria network.

3. Bicriteria path

A path π is a sequence $(I_1, \ldots, I_n)$ of two or more nodes $(n \geqslant 2)$, in which $(I_k, I_{k+1}) \in A$ for $k = 1, 2, \ldots, n-1$. Let P represent the set of all paths of the network and $p(j) = \{\pi \in P/I_1 = 0, I_n = j\}$ represent the set of all paths ending to node j from the source node.

Assume that for each arc $(i,j) \in A$ there is the value vector $l_{ij} = (\mu_{ij}, \sigma^2_{ij}) \in R^2$, in which μ_{ij} represents the mean and σ^2_{ij} represents the variance of the length of the arc. I define the path value function $z : P \to R^2$ in this manner: $z(\pi) = l_{i_1 i_2} o \ldots o l_{i_{n-1} i_n}$, in which o is a binary operator on R^2. This operator can be decomposed in the form $o = (+, +)$. Taking into account the mean and the variance as two indicated criteria, we have these equations for each $\pi \in P$:

$$\begin{aligned} z_1(\pi) &= \mu(\pi) = \mu_{i_1 i_2} + \cdots + \mu_{i_{n-1} i_n}, \\ z_2(\pi) &= \sigma^2(\pi) = \sigma^2_{i_1 i_2} + \cdots + \sigma^2_{i_{n-1} i_n}. \end{aligned} \tag{11}$$

Assume that $z(j) = \{z(\pi)/\pi \in p(j)\}$ is the set of all path value functions corresponding to all paths ending at node j from the source node. Let $z(1) = \{z_1\}$, in which $z_1 = (0,0)$. Therefore, $z_1 o z_{ij} = z_{ij}$ for all arcs $(i,j) \in A$.

Since the two indicated criteria (the mean and the variance) are in conflict with each other, I define the utility function $u : R^2 \to R$ on the set of path value functions in this manner:

$$u(\mu, \sigma^2) = \mu + \beta\sigma^2, \tag{12}$$

where β is a non-negative constant coefficient (see Keeney and Raiffa [8] for more details about the utility functions).

The objective is to minimize u subject to the path value functions corresponding to all paths ending at the sink node. Therefore, if b represents the sink node of the network, we face the following optimization problem:

$$\begin{aligned} &\text{Min} \quad f^* = u(z) \\ &\text{s.t.} \quad z \in z(b). \end{aligned} \tag{13}$$

4. Dynamic programming approach

The application of dynamic programming for solving multiple criteria problems with a specific utility function were considered in several papers. If we can prove that the utility function satisfies the monotonicity assumption, we can easily use dynamic programming for solving the problem (see Bellman [4] for more details).

Unfortunately, when we encounter a multicriteria utility function, the proof is not easy. We can extend the method of forward single criterion dynamic programming for obtaining the optimal solution, in this case. If we prove that the utility function satisfies the monotonicity assumption, we can obtain the shortest path by solving the following recursive equations:

$$\begin{aligned} &f(1) = z_1, \\ &f(j) = \arg\min u(z), \\ &z \in \{f(i) o l_{ij}/(i,j) \in A\} \quad j = 2, \ldots, b. \end{aligned} \tag{14}$$

Therefore, the best decision in node j is to find its best precedent node or $i^*(j)$, in which passing through arc $(i,j) \in A$ minimizes $u(z)$.

If we can prove the relation (15), the monotonicity assumption will be proved:

$$u(z) \leqslant u(\bar{z}) \Rightarrow u(z o l_{jk}) \leqslant u(\bar{z} o l_{jk}) \quad z, \bar{z} \in z(j) \ \forall j, k \in V, \text{ in which } (j,k) \in A. \tag{15}$$

This relation means that if we prefer the value vector z to the value vector $\bar{z}$, then we must prefer each extension of the path with the length z to the same extension of the path with the length $\bar{z}$.

Lemma 1. *The utility function* $u(\mu, \sigma^2) = \mu + \beta\sigma^2$ *satisfies the monotonicity assumption.*

Proof. Taking into account two indicated criteria and the utility function, without losing the generality of the problem, we can assume:

$$\begin{aligned} u(z) &= \mu + \beta\sigma^2, \\ u(\bar{z}) &= \bar{\mu} + \beta\bar{\sigma}^2, \\ l_{jk} &= (\mu_{jk}, \sigma^2_{jk}). \end{aligned} \tag{16}$$

Therefore, we have:

$$\begin{aligned} u(zol_{jk}) &= \mu + \mu_{jk} + \beta\sigma^2 + \beta\sigma^2_{jk} \\ u(\bar{z}ol_{jk}) &= \bar{\mu} + \mu_{jk} + \beta\bar{\sigma}^2 + \beta\sigma^2_{jk}. \end{aligned} \tag{17}$$

Considering:

$$u(z) = \mu + \beta\sigma^2 \leqslant \bar{\mu} + \beta\bar{\sigma}^2 = u(\bar{z}). \tag{18}$$

Then,

$$U(zol_{jk}) = \mu + \mu_{jk} + \beta\sigma^2 + \beta\sigma^2_{jk} \leqslant \bar{\mu} + \mu_{jk} + \beta\bar{\sigma}^2 + \beta\sigma^2_{jk} = u(\bar{z}ol_{jk}) \tag{19}$$

because the constant term $C = \mu_{jk} + \beta\sigma^2_{jk}$ was added to the both sides of inequality (18) and consequently the utility function satisfies the monotonicity assumption. □

The main steps of the proposed algorithm to find the bicriteria shortest path in the networks of queues are as follows:

Algorithm 1

Step 1. Transform the network of queues into a stochastic network by transforming each node containing a queueing system into a stochastic arc, whose length is equal to the waiting time in the corresponding queueing system.

Step 2. Transform the stochastic network into a bicriteria network by computing the mean and the variance of the waiting time in system for each service station and transferring these two criteria to the augmented arc of the network. The mean and the variance of the transport times between the service stations should also be computed in this step.

Step 3. Determine the value of β in the utility function (12).

Step 4. Solve the recursive equations (14), using the dynamic programming approach.

As the worst case, consider a network of queues with b service stations settled in all nodes of the network. We can compute Step 2 in O(b) operations. We can also compute Step 4 in O($2b - 1$) operations, because after transforming the network of queues into a bicriteria network, we need to compute $f(j)$ in the recursive functions (14) $2b - 1$ times, which is equal to the number of nodes in the bicriteria network minus 1. Therefore the time complexity of Algorithm 1 is O(b).

5. Numerical example

Consider the network of queues shown in Fig. 1.
The assumptions are as follows:

1. There is only one external arrival process to node 1 according to a Poisson process with the average of 0.8 per hour ($\lambda = 0.8$).
2. There is a service station with infinite servers in node 1 whose distribution of service time is normal with parameters $(\mu, \sigma^2) = (10, 1)(B'(t) = N(10, 1))$.

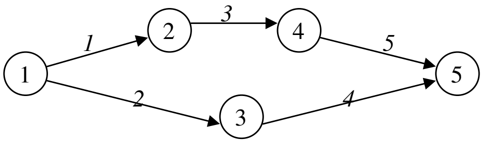

Fig. 1. Example of a network of queues.

3. There is a service station with one server in node 2 whose distribution of service time is gamma with parameters $(\alpha, \beta = 2, 1)$ $(B'(t) = te^{-t}\ t > 0)$.
4. There is a service station with one server in node 3 whose distribution of service time is exponential with parameter $(\mu = 0.6)$ $(B'(t) = 0.6e^{-0.6\ t}t > 0)$.
5. There is a service station with one server in node 4 whose distribution of service time is exponential with parameter $(\mu = 1)$ $(B'(t) = e^{-\ t}t > 0)$.
6. There is no service station in node 5.
7. The arc lengths $(1,2)$, $(1,3)$, $(2,4)$, $(4,5)$ are 0.
8. The distribution of the length of arc $(3,5)$ is normal with parameters $(\mu, \sigma^2) = (4,2)$.
9. The departure process from node 1 goes to one of the service stations settled in node 2 or 3 with equal probabilities.

Now, the objective is to find the shortest path from node 1 to node 5 in this network of queues. For this purpose, the network of queues is transformed into a bicriteria network shown in Fig. 2, using Algorithm 1. The following illustrations are related to this bicriteria network:

1. The vector corresponding to arc $(1,2)$ indicates the mean and the variance of the waiting time in system in $M/G/\infty$ settled in node 1 of the original network.
2. The vector corresponding to arc $(2,4)$ indicates the mean and the variance of the waiting time in system in $M/G/1$ settled in node 2 of the original network, which are computed as follows:

$$\begin{aligned} &B^*(s) = \int_0^\infty e^{-st}te^{-t}\,dt = 1/(1+s)^2, \\ &W^*(s) = 0.2/((1+s)^2 - 0.4s - 0.8), \\ &\mu^* = -dW^*(s)/ds|_{s=0} = 8, \\ &\sigma^{*2} = d^2W^*(s)/ds^2|_{s=0} - \mu^{*2} = 118 - 64 = 54. \end{aligned} \tag{20}$$

3. The vector corresponding to arc $(2,3)$ indicates the mean and the variance of the waiting time in system in $M/M/1$ settled in node 3 of the original network, which are computed as follows:

$$\begin{aligned} &\mu^* = (1/(\mu - \lambda)), \\ &\sigma^{*2} = (1/(\mu - \lambda))^2, \\ &\mu = 0.6, \quad \lambda = 0.4, \\ &\mu^* = 5, \\ &\sigma^{*2} = 25. \end{aligned} \tag{21}$$

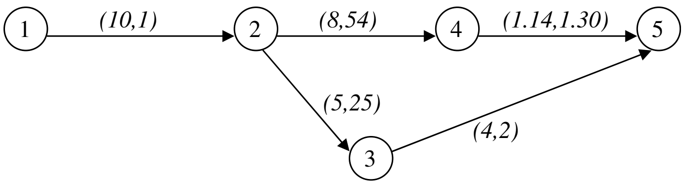

Fig. 2. The bicriteria network corresponding to the network of queues.

4. The vector corresponding to arc $(4,5)$ indicates the mean and the variance of waiting time in system in $G/M/1$ settled in node 4 of the original network, which are computed as follows (in this case, the density function of the time between two successive departures from node 2 or $C'(t)$ is equal to the density function of the time between two successive arrivals at node 4 or $A'(t)$, because customers after passing node 2 of the original network will arrive at node 4 immediately):

$$\begin{aligned}
&A'(t) = \rho B'(t) + (1-\rho)\int_0^t B'(t-u)\lambda e^{-\lambda u}\,du,\\
&B'(t) = te^{-t},\\
&\lambda = 0.4, \quad \rho = 0.8,\\
&A'(t) = 0.8te^{-t} + 0.08\int_0^t (t-u)e^{-(t-u)}e^{-0.4u}\,du,\\
&A'(t) = 0.67te^{-t} + 0.22e^{-0.4t} - 0.22e^{-t}.
\end{aligned} \tag{22}$$

After computing $A'(t)$ and knowing that the service rate of $G/M/1$ is equal to 1, we can compute x_0 from the following equation:

$$\begin{aligned}
z &= \int_0^\infty e^{-t(1-z)}(0.67te^{-t} + 0.22e^{-0.4t} - 0.22e^{-t})\,dt, \quad 0 < z < 1, z\\
&= 0.67/(2-z)^2 + 0.22/(1.4-z) - 0.22/(2-z), \quad z = 0.1225.
\end{aligned} \tag{23}$$

As mentioned, we can compute the mean and the variance of the waiting time in the system from the following equations:

$$\begin{aligned}
&\mu^* = 1/\mu(1-x_0),\\
&\sigma^{*2} = (1/\mu(1-x_0))^2,\\
&\mu = 1,\\
&x_0 = 0.1225,\\
&\mu^* = 1.14,\\
&\sigma^{*2} = 1.30.
\end{aligned} \tag{24}$$

5. The vector corresponding to arc $(3,5)$ indicates the mean and the variance of the length of arc $(3,5)$ in the original network.

Assume that the utility function is in this form:

$$u(\mu, \sigma^2) = \mu + 0.2\sigma^2. \tag{25}$$

Table 1 shows the best path value function or $f(j)$ and the value of the related utility function or $u(f)$ and also the best decision (precedent node) or $i^*(j)$ for each node j, which are obtained by solving the recursive equations (14).

Table 1
The results of the numerical example

Node j	$f(j)$	$u(f)$	$i^*(j)$
1	$(0,0)$	0	_
2	$(10,1)$	10.2	1
3	$(15,26)$	20.2	2
4	$(18,55)$	29	2
5	$(19,28)$	24.6	3

Therefore, the optimal path in the bicriteria network is 1-2-3-5 corresponding to the path 1-3-5 of the network of queues. This path is a sufficient path, because it dominates the second path of the network in both criteria. The mean and the variance of this path are

$$\begin{aligned} \mu^* &= 19, \\ \sigma^{*2} &= 28. \end{aligned} \tag{26}$$

6. Conclusion

In this paper, I developed a polynomial algorithm to find the shortest path from the source to the sink node of a network of queues in the steady-state, based on queueing theory, dynamic programming, graph theory and also multiple criteria decision making. It was assumed that some nodes of the network contain service stations including either one or infinite servers with general distributions of service time. Moreover, the arc lengths among the service stations are assumed to be independent random variables with general distribution functions. I also proved that the defined utility function satisfies the monotonicity assumption that is essential for applying the dynamic programming for solving this bicriteria network problem.

The proposed algorithm is useful to solve the problem of routing a ship in ocean considering the waiting time in each region for anchoring and fuelling, as mentioned in Section 1.

The proposed methodology can also be applied to approximate the mean and the variance of project completion time in dynamic Markov PERT networks, where the activity durations are exponentially distributed random variables and the new projects are generated according to a Poisson process (see Azaron and Tavakkoli-Moghaddam [3] for details). In this case, we just need to replace min with max in the recursive equations (14).

Even if there are parallel service stations like $M/M/C$ and $G/M/C$ in the network, we can still apply Algorithm 1, because the distribution functions of the waiting time in system in these service stations are available (see Gross and Harris [7] for more details). Therefore, after computing the mean and the variance of the waiting time in system in those service stations, the proposed algorithm is applied.

Acknowledgements

This research is supported by Science Foundation Ireland under Grant No. 03/CE3/I405 as part of the Centre for Telecommunications Value-Chain-Driven Research (CTVR) and under Grant No. 00/PI.1/C075.

References

[1] A. Azaron, F. Kianfar, Dynamic shortest path in stochastic dynamic networks: ship routing problem, European Journal of Operational Research 144 (2003) 138–156.
[2] A. Azaron, M. Modarres, Distribution function of the shortest path in networks of queues, OR Spectrum 27 (2005) 123–144.
[3] A. Azaron, R. Tavakkoli-Moghaddam, A multi-objective resource allocation problem in dynamic PERT networks, Applied Mathematics and Computation, in press, doi:10.1016/j.amc.2006.01.027.
[4] R. Bellman, Dynamic Programming, Princeton University Press, 1957.
[5] R. Carraway, T. Morin, H. Moskowitz, Generalized dynamic programming for multicriteria optimization, European Journal of Operational Research 44 (1990) 95–104.
[6] J. Current, H. Min, Multiobjective design of transportation networks: taxonomy and annotation, European Journal of Operational Research 26 (1986) 187–201.
[7] D. Gross, M. Harris, Fundamentals of Queueing Theory, second ed., J. Wiley, 1985.
[8] R. Keeney, H. Raiffa, Decisions with Multiple Objectives: Preferences and Value Tradeoffs, J. Wiley, 1976.
[9] H. Lee, S. Pulat, Bicriteria network flow problems: continuous case, European Journal of Operational Research 51 (1991) 119–126.
[10] E. Martins, On a multi-criteria shortest path problem, European Journal of Operational Research 16 (1984) 236–245.
[11] A. Wijerante, M. Turnquist, P. Mirchandani, Multiobjective routing of hazardous materials in stochastic networks, European Journal of Operational Research 65 (1993) 33–43.