OR Spectrum
DOI 10.1007/s00291-006-0058-z

REGULAR ARTICLE

Amir Azaron · Hideki Katagiri · Kosuke Kato · Masatoshi Sakawa

A multi-objective discrete reliability optimization problem for dissimilar-unit standby systems

Abstract A new methodology for the reliability optimization of a *k* dissimilar-unit nonrepairable cold-standby redundant system is introduced in this paper. Each unit is composed of a number of independent components with generalized Erlang distributions of lifetimes arranged in a series–parallel configuration. We also propose an approximate technique to extend the model to the general types of nonconstant hazard functions. To evaluate the system reliability, we apply the shortest path technique in stochastic networks. The purchase cost of each component is assumed to be an increasing function of its expected lifetime. There are multiple component choices with different distribution parameters available for replacement with each component of the system. The objective of the reliability optimization problem is to select the best components, from the set of available components, to be placed in the standby system to minimize the initial purchase cost of the system, maximize the system mean time to failure, minimize the system variance of time to failure, and also maximize the system reliability at the mission time. The goal attainment method is used to solve a discrete time approximation of the original problem.

Keywords Reliability optimization · Stochastic networks · Shortest path · Optimal control

JEL Classification D81 · D85 · C61 · C63 · C65

H. Katagiri · K. Kato · M. Sakawa
Department of Artificial Complex Systems Engineering, Graduate School of Engineering, Hiroshima University, Kagamiyama 1-4-1, Higashi-Hiroshima, Hiroshima, 739-8527, Japan

A. Azaron (✉)
Department of Computer Science, Cork Constraint Computation Centre,
University College Cork, Cork, Ireland
E-mail: a.azaron@4c.ucc.ie

1 Introduction

Many fielded systems use cold-standby redundancy as an effective system design strategy. Cold-standby means that the redundant units cannot fail while they are waiting. Space exploration and satellite systems achieve high reliability by using cold-standby redundancy for nonrepairable systems (Sinaki 1994). Space inertial reference units are required to accurately monitor critical information for extended mission times without opportunities for repair. Many other systems use cold-standby redundancy as an effective strategy to achieve high reliability including textile manufacturing systems (Pandey et al. 1996) and carbon recovery systems used in fertilizer plants (Kumar et al. 1996).

Extensive research has been carried out on the reliability of redundant systems with similar/dissimilar units. Several methods and methodologies have been discussed by Birolini (1994) and Srinivasan and Subramanian (1980).

Multicomponent systems were analyzed by several authors including Goel et al. (1983) and Yamashiro (1981). Most of such studies deal with the analysis of a single unit system. Gupta et al. (1986) investigated a single server, two-unit multicomponent cold-standby system under the assumption that the cold-standby unit becomes operative instantaneously upon the failure of operative unit. Gupta et al. (1997) analyzed a two dissimilar-unit multicomponent cold-standby system with correlated failures and repairs.

There are few researches about k dissimilar-unit multicomponent systems because of the complexity in the equations and not getting the results in closed form. Azaron et al. (2005a) developed a new approach to evaluate the reliability function of a class of dissimilar-unit redundant systems with exponentially distributed lifetimes.

In this paper, we present a new methodology for the reliability optimization of a k dissimilar-unit multicomponent, nonrepairable cold-standby redundant system. Each unit is composed of a number of independent components arranged in a series–parallel configuration. The components' lifetimes are assumed to be independent random variables with generalized Erlang distributions. Therefore, this methodology allows nonconstant hazard functions. We also propose an approximate technique to extend the model to the case of general lifetime distributions.

The purchase cost of each component is assumed to be an increasing function of its expected lifetime. In other words, it is possible to increase the expected lifetime of each component by placing a more expensive unit in the system. There is a set of component choices with different distribution parameters eligible to be replaced with each component of the system. The problem is to select the best components from these sets. This problem is formulated as a multi-objective discrete optimal control problem that involves four conflicting objective functions. The objective functions are the total costs of the standby system (to be minimized), the mean time to failure of the system (max), the variance of the system lifetime (min), and the system reliability at the given mission time (max). This approach involves the use of graph theory, Markov processes, reliability analysis, and multiple objective programming.

There are few researches toward the reliability optimization of nonrepairable systems with cold-standby redundancy scheme. The problem has often been solved for nonrepairable active redundant systems using dynamic programming (Fyffe et al. 1968; Nakagawa and Miyazaki 1981) and integer programming (Bulfin and Liu

1985; Gen et al. 1990). Gnedenko and Ushakov (1995) presented algorithms to maximize the median time to failure. Nakashima and Yamato (1977) solved an analogous problem to maximize the time period where system reliability remains above a preselected value. Their algorithm assumes that components have exponential lifetimes, but that the distribution parameters are the decision variables to be determined in addition to the redundancy levels.

The problem of reliability optimization of nonrepairable cold-standby redundant systems has received less attention. Albright and Soni (1984) have solved a reliability optimization problem for nonrepairable systems with standby redundancy. They assumed exponential lifetime and one component choice per subsystem. Robinson and Neuts (1989) studied system design for nonrepairable systems with cold-standby redundancy. They considered systems with components that have phase-type lifetime distributions. Coit (2001) has determined optimal design configurations for nonrepairable series–parallel systems with cold-standby redundancy. His problem formulation considers nonconstant component hazard functions and imperfect switching. Prasad et al. (1999) considered the problem of allocating multifunctional redundant components for deterministic and stochastic mission times. In their formulation, there is a limit on the total number of redundant components, which can be used.

There are also a few papers that consider the multi-objective reliability optimization for either time-independent case (see Sakawa 1978) or active redundant systems (Sakawa 1980; Dhingra 1992) and optimize system reliability, cost, weight, and volume for a given mission time. Azaron et al. (2005b) used the surrogate worth trade-off method to find the optimal distribution parameters (continuous decision variables like Nakagawa and Miyazaki 1981) in a cold-standby system.

The major limitations in the reliability evaluation and optimization approaches for dissimilar-unit cold-standby systems, thus far, are:

1. Most available algorithms assume that each unit is composed of a single component, but they also cannot get the results in closed form (Goel and Gupta 1983).
2. Available algorithms that do address dissimilar-unit multicomponent cold-standby systems assume that each unit is composed of a number of components arranged in a series configuration. Although this is a start, there are many more complicated system configurations that should be examined. The problem lies in the difficulty of presenting more complicated structures.
3. Existing system reliability optimization algorithms are most often available for active redundancy. The logarithm of system reliability for an active standby redundant system is a separable function; dynamic programming or integer programming can be used to determine optimal solutions to the problem.
4. Available algorithms that do address cold-standby optimization generally assume similar redundant units and exponential lifetimes.
5. Most available optimization algorithms consider continuous decision variables. In this case, it is difficult in practice to select a component to match a specific distribution parameter.
6. Only one criterion for time-dependent reliability, like maximizing mean time to failure (MTTF) or maximizing the system reliability at a given mission time is considered in the model. In the reliability optimization problem, one often wishes to lower the risk that systems with short system lifetime are produced, but only maximizing MTTF is not always fit for the requirement, especially

when the optimally designed system has a large variance of time to failure (VTTF). The system reliability at the mission time is another important criterion, which should be considered in the model.

This paper not only considers the reliability optimization for a complex structure (dissimilar-unit cold-standby system, in which each unit is composed of a number of independent components with nonconstant hazard functions arranged in a series–parallel configuration), but also, the system is optimized with respect to the four important conflicting objectives.

We formulate the appropriate multi-objective discrete optimal control problem, in which the decision variables are the distribution parameters so that they are to be determined from some discrete sets. The problem formulation is continuous time, combinatorial, and stochastic. We prove that solving the resulting problem by standard optimal control techniques is impossible. Therefore, we do the discretization of time and convert the discrete optimal control problem into an equivalent mixed-integer nonlinear optimization problem. Finally, we use the goal attainment technique to solve this new multi-objective problem.

The remainder of this paper is organized in the following way. In Section 2, we extend the work of Azaron et al. (2005a) to evaluate the reliability function of a k dissimilar-unit multicomponent cold-standby redundant system. In Section 3, we present the multi-objective discrete reliability optimization problem. Section 4 presents the computational experiments, and finally we draw the conclusion of the paper in Section 5.

2 Reliability evaluation of dissimilar-unit nonrepairable cold-standby systems

A very efficient method to compute the reliability of a system is to express it as a reliability graph (see Shooman 1991 for the details). Reliability graphs consist of a set of arcs. Each arc represents a component of the system, while the nodes of the graph tie the arcs together and form the structure. Corresponding with the ith arc of the reliability graph, i=1,2,...,n, there is a random variable T_i as the lifetime of the ith component with generalized Erlang distribution of order n_i with parameters $\lambda_{i_1}, \lambda_{i_2}, \ldots, \lambda_{i_{n_i}}$. An Erlang distribution of the order n_i is a generalized Erlang distribution with $\lambda_{i_1} = \lambda_{i_2} = \ldots = \lambda_{i_{n_i}}$. When n_i=1, the underlying distribution becomes exponential with parameter λ_{i_1}. T_i, i=1,2,..., n, are independent random variables, due to the fact that the components work independently.

By definition, a cut of the graph is a set of arcs, which interrupts all connections between input and output when removed from the graph. A minimal cut is the one that contains no other cuts within it. Each system failure can be represented by the removal of at least one minimal cut from the graph.

As mentioned before, we consider a dissimilar-unit cold-standby system, where each unit is composed of a number of components with series–parallel configuration and not all of its components are set to function at time zero. Initially, only the components of the first path of the reliability graph work. Upon failing one component of this path, the system is switched to the next path and the connection between the input and the output is established through this second path. This process continues until no more connection between the input and the output of the graph exists. In that case, the system fails. In the systems, which we discuss in this paper, the minimal cuts are not coincided with the paths of the reliability graph.

Notations

T_i	Lifetime of the ith component of the system, i=1,2,...,n
T	System lifetime
C_j	jth minimal cut of the reliability graph, j=1,2,...,m
P_j	jth paths in the directed network
X_j	Failure time of the jth minimal cut of the reliability graph, j=1,2,...,m
$R(t)$	Reliability function of the system
$F(t)$	Distribution function of shortest path, from the source to the sink node, in directed network

Lemma 1 For j=1,2,...,m, the following relation holds:

$$X_j = \sum_{i \in C_j} T_i \tag{1}$$

Proof Taking into account the cold-standby nature of the structure, upon failure of each component of the jth minimal cut, the system is switched to the next path. Since this minimal cut does not coincide with any path of the reliability graph, then at any moment only one component of the jth minimal cut is activated. Therefore, the failure time of this cut is the sum of all its components.

To evaluate the reliability function, we construct a directed stochastic network with exponentially distributed arc lengths. There are m paths in this network, in which the jth path of this directed network corresponds with the jth minimal cut of the reliability graph j=1,2,...,m. Clearly, by Lemma 1, the length of each path in this directed network is equal to the failure time of the corresponding cut. In constructing this network, we use the idea that if the lifetime of the ith component of the system is distributed according to a generalized Erlang distribution of order n_i with parameters $\lambda_{i_1}, \lambda_{i_2}, \ldots, \lambda_{i_{n_i}}$, it can be decomposed to n_i exponential serial arcs with parameters $\lambda_{i_1}, \lambda_{i_2}, \ldots, \lambda_{i_{n_i}}$. The following rule describes how to construct the proper directed network.

Rule 1 Arc i belongs to the jth path of the directed network, if and only if $i \in C_j$. If n_i=1, then the length of this arc would be exponentially distributed with parameter λ_{i_1}. Otherwise, if n_i>1, then this arc is substituted with n_i exponential serial arcs with parameters $\lambda_{i_1}, \lambda_{i_2}, \ldots, \lambda_{i_{n_i}}$.

Example 1 To operate the accounting activities of a firm, either one computer or one calculator is needed. The calculator needs one battery to do the required operations. However, there are two batteries available in the system to function as standby. At the beginning, the system may start with the computer. If it fails, then the calculator with one battery is doing the necessary operations. In that case, if the calculator fails, so does the system. However, if the battery fails, the calculator works with the standby one. In fact, if either calculator or the second battery fails, then the operation comes to the end.

The system can be represented by a reliability graph, as depicted in Fig. 1, in which arc 1 represents the computer, arc 2 represents the calculator, arc 3 and arc 4 represent the first and the second battery, respectively.

This is an example of a two dissimilar unit multicomponent cold-standby system. The first unit is the computer, but the second unit is composed of a calculator and two batteries. It is assumed that the lifetimes of elements in this example are all exponentially distributed.

This reliability graph has three paths. P_1=(1) corresponds with the first active unit (computer), while P_2=(2,3) and P_3=(2,4) are two paths corresponding with the second unit. Even if we change the order of paths corresponding with the second unit, the final result will not change, because of the memoryless property of the lifetimes of elements. Two minimal cut sets of the reliability graph are C_1=(1,2) and C_2=(1,3,4). From Lemma 1, the failure times of the minimal cuts are

$$X_1 = T_1 + T_2,$$
$$X_2 = T_1 + T_3 + T_4.$$

Therefore, we construct the directed network following Rule 1, as depicted in Fig. 2. This network has two paths, P_1=(1, 2) and P_2=(1,3,4).

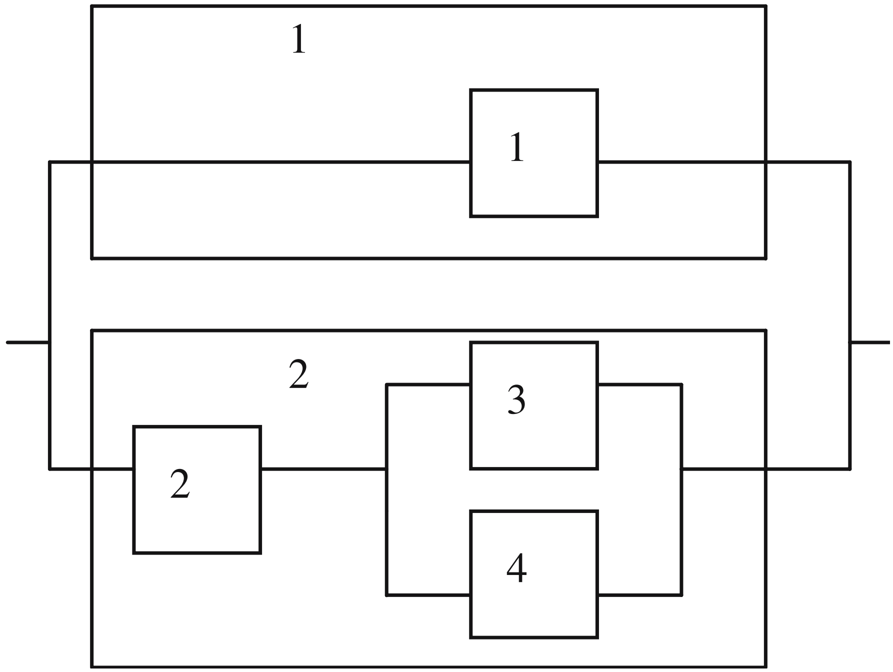

Fig. 1 Reliability graph of Example 1

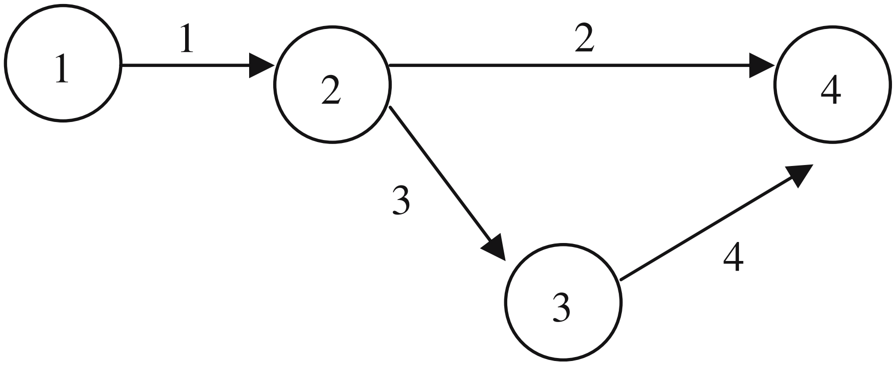

Fig. 2 Directed network of Example 1

Theorem 1 The system lifetime is given by

$$T = \min_{j=1,2,\ldots,m} \{X_j\}. \tag{2}$$

Proof Upon the failure of the first minimal cut of the reliability graph of the system, all connections between the input and the output are interrupted and, consequently, the system fails. Therefore, the lifetime of the system would be equal to the failure time of the first minimal cut, which results in Eq. 2.

Corollary 1 The reliability function of the system is given by

$$R(t) = 1 - F(t). \tag{3}$$

Proof Relation 3 follows from the definitions of $R(t)$ and $F(t)$.

2.1 Shortest path analysis in directed networks

Kulkarni's method (Kulkarni 1986) is applied to obtain the distribution function of the shortest path from the source to the sink node in the directed network and, accordingly, the reliability function of the cold-standby system.

Let $G=(V, A)$ be a directed network, in which V and A represent the sets of nodes and arcs of the network, respectively. Let s and t represent the source and the sink nodes of this network, respectively. The length of arc $(u,v)\in A$ is indicated by $T_{(u,v)}$, which is an exponential random variable with parameter $\lambda_{(u,v)}$.

For constructing the proper stochastic process, it is convenient to visualize the stochastic network as a communication network with the nodes as stations capable of receiving and transmitting messages and arcs as one-way communication links connecting pairs of nodes. The messages are assumed to travel at a unit speed so that $T_{(u,v)}$ denotes the travel time from node u to v. As soon as a node receives a message over one of the incoming arcs, it transmits it along all the outgoing arcs and then disables itself, i.e., it loses the ability to receive and transmit the future messages. This process continues until the message reaches the sink node t. Now, at any time there may be some nodes and arcs in the stochastic network that are "useless" for the progress of the message toward the sink node, i.e., even if the messages are received and transmitted by these nodes and carried by these arcs, the message can only reach disabled nodes. It is assumed that all such useless nodes are also disabled, and the messages traveling on such arcs are aborted. Now, let $X(t)$ be the set of all disabled nodes at time t. $X(t)$ is called the state of the network at time t.

Definition 1 To describe the evolution of the stochastic process $\{X(t),\ t\geq 0\}$, for each $X\subset V$, where $s\in X$ and $t\in \overline{X}=V-X$, we define the following sets:

1. $\overline{X}_1\subset\overline{X}$, set of nodes not included in X with the property that each path that connects any node of this set to the sink node t, contains at least one member of X.
2. $S(X)=X\cup\overline{X_1}$

Definition 2

$$\begin{aligned}\Omega &= \{X\subset V/s\in X,\ t\in\overline{X},\ X=S(X)\},\\ \Omega^* &= \Omega\cup V.\end{aligned} \tag{4}$$

Example 2 In the directed network depicted in Fig. 3, if we consider X=(1,2), then $\overline{X}_1=\phi$, and $S(X)$=(1,2). However, if we consider X=(1,3,4), then the only path that connects node (2) $\in\overline{X}$ to node (5) passes through node (4), which belongs to X. Therefore, $\overline{X}_1=(2)$, and $S(X)$=(1,2,3,4). In this example, Ω^*={(1),(1,2),(1,3),(1,2,3),(1,2,4),(1,2,3,4),(1,2,3,4,5)}.

Definition 3 If $X\subset V$ such that $s\in X$ and $t\in\overline{X}$, then a cut is defined as:

$$C(X,\overline{X})=\{(u,v)\in A/u\in X,v\in\overline{X}\}. \tag{5}$$

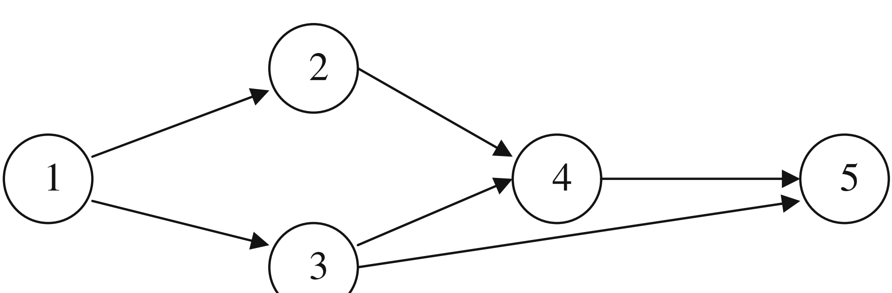

Fig. 3 Directed network of Example 2

There is a unique minimal cut contained in $C(X,\overline{X})$, denoted by $C(X)$. If $X \in \Omega$ then,

$$C(X,\overline{X}) = C(X).$$

It is shown that $\{X(t), t \geq 0\}$ is a continuous time Markov process with state space Ω^* and the infinitesimal generator matrix $Q = [q(X,Y)](X, Y \in \Omega^*)$ (see Kulkarni 1986 for the details), where

$$q(X,Y) = \begin{cases} \sum_{(u,v)\in C(X)} \lambda_{(u,v)} & \text{if } Y = S(X \cup \{v\}), \\ -\sum_{(u,v)\in C(X)} \lambda_{(u,v)} & \text{if } Y = X, \\ 0 & \text{otherwise}. \end{cases} \quad (6)$$

We assume that the states in Ω^* are numbered $1, 2, \ldots, N = |\Omega^*|$ so that Q matrix is upper triangular. State 1 is the initial state, and state N is the final (absorbing) state. In example 2, state 1 is (1), and state 7 is (1,2,3,4,5).

Let T represent the length of the shortest path in the directed network. Clearly,

$$T = \min\{t > 0 : X(t) = N / X(0) = 1\}. \quad (7)$$

Therefore, the length of the shortest path in the directed network would be equal to the time until $\{X(t), t \geq 0\}$ gets absorbed in the final state N, starting from state 1.

Chapman–Kolmogorov backward equations can be applied to compute $F(t)=P\{T \leq t\}$. If we define:

$$P_i(t) = P\{X(t) = N / X(0) = i\} i = 1, 2, \ldots, N, \quad (8)$$

then, $F(t)=P_1(t)$.

The system of differential equations for the vector $P(t)=[P_1(t), P_2(t), \ldots, P_N(t)]^T$ is given by

$$\begin{aligned} \dot{P}(t) &= QP(t), \\ P(0) &= [0, 0, \ldots, 1]^T, \end{aligned} \quad (9)$$

where $P(t)$ represents the state vector of the system, and Q is the infinitesimal generator matrix. By taking advantage of the upper triangular nature of Q, the differential Eq. 9 can be easily solved. After computing $F(t)$, the system reliability can be computed from Eq. 3.

3 Multi-objective discrete reliability optimization problem

In this section, we develop a multi-objective discrete model to select the best components from the set of available components to be placed in the cold-standby system. In fact, we may increase the expected lifetime of each component by placing a more expensive component in the system. In that case, the mean time to failure of the system will be increased. However, it clearly causes the initial purchase cost of the system to be increased, accordingly. Consequently, an appropriate trade-off between cost and reliability is required.

To achieve the above-mentioned goals, we developed a multi-objective problem, in which four objectives are sought simultaneously, minimizing initial purchase cost, maximizing MTTF, minimizing VTTF, and also maximizing system reliability $R(u)$ at the given mission time, u.

The purchase cost of each component is assumed to be an increasing function of its expected lifetime. The expected lifetime of a component with generalized Erlang distribution of the order n_i and the parameters $\left(\lambda_{i_1}, \lambda_{i_2}, \ldots, \lambda_{i_{n_i}}\right)$ is equal to $\sum_{j=1}^{n_i} \frac{1}{\lambda_{i_j}}$. Therefore, C or the initial purchase cost of the standby system is given by

$$C = \sum_{i=1}^{n} g_i \left(\sum_{j=1}^{n_i} \frac{1}{\lambda_{i_j}} \right) \tag{10}$$

where $g_i\left(\sum_{j=1}^{n_i} \frac{1}{\lambda_{i_j}}\right)$ is an increasing function with respect to $\sum_{j=1}^{n_i} \frac{1}{\lambda_{i_j}}$.

MTTF and VTTF are given by

$$\text{MMTF} = \int_0^\infty (1 - P_1(t))dt, \tag{11}$$

$$\text{VTTF} = \int_0^\infty t^2 \dot{P}_1(t)dt - \left[\int_0^\infty t\dot{P}_1(t)dt\right]^2. \tag{12}$$

Considering S_{i_j} as the set of different values of $\lambda_{i_j}\left(\lambda_{i_j} \in S_{i_j}\right)$ corresponding with the jth distribution parameter of available functionally equivalent components eligible to be replaced with the ith component, the infinitesimal generator matrix Q would be a function of the control vector $\lambda = \left[\lambda_{i_j},\ i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n_i\right]^T$. Therefore, the dynamic model would be

$$\begin{gathered} \dot{P}(t) = Q(\lambda)P(t), \\ P_i(0) = 0 \quad i = 1, 2, \ldots, N-1, \\ P_N(t) = 1. \end{gathered} \tag{13}$$

Considering R as the system reliability at the mission time u, the appropriate multi-objective discrete optimal control problem is

$$\begin{aligned}
&\text{Min } C = \sum_{i=1}^{n} g_i \left(\sum_{j=1}^{n_i} \frac{1}{\lambda_{i_j}} \right) \\
&\text{Max MTTF} = \int_0^\infty (1 - P_1(t)) dt \\
&Min\ VTTF = \int_0^\infty t^2 \dot{P}_1(t) \mathrm{d}t - \left[\int_0^\infty t \dot{P}_1(t) \mathrm{d}t \right]^2 \\
&\text{Max } R = 1 - P_1(u) \\
&s.t. \\
&\dot{P}(t) = Q(\lambda) P(t) \\
&P_i(0) = 0 \quad i = 1, 2, \ldots, N-1 \\
&P_N(t) = 1 \\
&\lambda_{i_j} \in S_{i_j} \quad i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n_i
\end{aligned} \tag{14}$$

We try to solve problem 14 optimally, using the Maximum Principle (for the details, see Sethi and Thompson 1981). For simplicity, we consider only one objective function, for example $MTTF = \int_0^\infty (1 - P_1(t)) \mathrm{d}t$, in the model.

Considering S as the set of allowable controls, which consists of the last set of constraints of problem 14 ($\lambda \in S$), and N vector $\mu(t)$ as the adjoint vector function, the Hamiltonian function would be

$$H(\mu(t), P(t), \lambda) = \mu(t)^T Q(\lambda) P(t) + 1 - P_1(t). \tag{15}$$

In Eq. 15, $\mu(t)$ plays the role of Lagrange multipliers in nonlinear optimization, but in optimal control theory. Then, we write the adjoint equations and terminal conditions, which are

$$\begin{aligned}
-\dot{\mu}(t)^T &= \mu(t)^T Q(\lambda) + [-1, 0, \ldots, 0], \\
\mu(T)^T &= 0, \quad T \to \infty.
\end{aligned} \tag{16}$$

If we could compute $\mu(t)$ from Eq. 16, we could maximize the Hamiltonian function subject to $\lambda \in S$ to get the optimal control λ^*, and we could solve the problem optimally. Unfortunately, the adjoint equations (16) are dependent on the unknown control vector λ; therefore, they cannot be solved directly.

If we could also maximize the Hamiltonian function (Eq. 15), subject to $\lambda \in S$, for an optimal control function in closed form as $\lambda^* = f(P^*(t), \mu^*(t))$ then we could substitute this into the state equations, $\dot{P}(t) = Q(\lambda)P(t),\ P(0) = [0, 0, \ldots, 1]^T$, and adjoint equations (16) to get a set of differential equations, which is a two-point boundary value problem. Unfortunately, we cannot obtain λ^* by differentiating H with respect to λ, because λ is a discrete vector and, consequently, λ^* cannot be obtained in closed form.

According to the two mentioned points, it is impossible to solve the optimal control problem 14 optimally, even in the case of single objective problem. Relatively few optimal control problems can be solved optimally. Therefore, we try to solve this problem, approximately. To do that, we do the discretization of time and convert the multi-objective discrete optimal control problem into an equivalent multi-objective mixed integer nonlinear programming one. In other words, we transform the differential equations to the equivalent difference equations and transform the integral terms into equivalent summation terms. To follow this approach, the time interval is divided into K equal portions with the length of Δt. If Δt is sufficiently small, it can be assumed that $P(t)$ varies only in times $0, \Delta t, \ldots, (K-1)\Delta t$.

Consider $P(k\Delta t)$ as $P(k)$, the continuous time system $\dot{P}(t) = Q(\lambda)P(t)$ is approximated as the following discrete time system:

$$P(k+1) = P(k) + Q(\lambda)P(k)\Delta t \quad k = 0, 1, \ldots, K-1. \tag{17}$$

Similarly, MTTF and VTTF are approximated as:

$$\mathrm{MTTF}_a = \sum_{k=0}^{K} (1 - P_1(k))\Delta t, \tag{18}$$

$$\mathrm{VTTF}_a = \sum_{k=0}^{K-1} (k\Delta t)^2 (P_1(k+1) - P_1(k)) - \left[\sum_{k=0}^{K-1} k\Delta t (P_1(k+1) - P_1(k))\right]^2 \tag{19}$$

Since each $P_i(k)$ for i=1,2,..., $N-1$, k=1,2,...,K is a distribution function, then we should consider the following constraints in the discrete time approximation problem.

$$P_i(k) \leq 1 \quad i = 1, 2, \ldots, N-1, \; k = 1, 2, \ldots, K. \tag{20}$$

3.1 Goal attainment method

This method requires setting up a goal and weight, b_j and c_j ($c_j \geq 0$) for j=1,2,3,4, for the four indicated objective functions. c_j relates the relative under-attainment of the b_j. For under-attainment of the goals, a smaller c_j is associated with the more important objectives. c_j, j=1,2,3,4, are generally normalized so that $\sum_{i=1}^{4} c_j = 1$.

Considering $\left[\frac{u}{\Delta t}\right]$ as the integer part of $\frac{u}{\Delta t}$, the appropriate goal attainment formulation would be

$$\begin{aligned}
&Min\, z\\
&s.t:\\
&\sum_{i=1}^{n} g_i\left(\sum_{j=1}^{n_i}\frac{1}{\lambda_{i_j}}\right) - c_1 z \le b_1\\
&\sum_{k=0}^{K}(1-P_1(k))\Delta t + c_2 z \ge b_2\\
&\sum_{k=0}^{K-1}(k\Delta t)^2(P_1(k+1)-P_1(k)) - \left[\sum_{k=0}^{K-1}k\Delta t(P_1(k+1)-P_1(k))\right]^2 - c_3 z \le b_3\\
&1 - P_1\left(\left[\frac{u}{\Delta t}\right]\right) + c_4 z \ge b_4\\
&P(k+1) = P(k) + Q(\lambda)P(k)\Delta t \quad k = 0, 1, \ldots, K-1\\
&P_i(0) = 0 \quad i = 1, 2, \ldots, N-1\\
&P_N(k) = 1 \quad k = 0, 1, \ldots, K\\
&P_i(k) \le 1 \quad i = 1, 2, \ldots, N-1,\ k = 1, 2, \ldots, K\\
&\lambda_{i_j} \in S_{i_j} \qquad i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n_i\\
&z \ge 0
\end{aligned} \tag{21}$$

The optimal solution using this formulation is fairly sensitive to b and c. Depending upon the values for b, it is possible that c does not appreciably influence the optimal solution. Instead, the optimal solution can be determined by the nearest Pareto-optimal solution from b. This might require that c be varied parametrically to generate a set of Pareto-optimal solutions.

For solving the goal attainment formulation (Eq. 21), we define the new 0–1 decision variables y_{ijk}. Let α_{ijk} represent the kth member of S_{i_j}, $k = 1, 2, \ldots, \left|S_{i_j}\right|$. Then, $\lambda_{i_j} \in S_{i_j}, i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n_i$, in Eq. 21 should be replaced with the constraints 22 and 23.

$$\lambda_{ij} = \sum_{k=1}^{\left|S_{ij}\right|}\alpha_{ijk}y_{ijk} \quad i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n_i. \tag{22}$$

$$\sum_{k=1}^{\left|S_{i_j}\right|} y_{ijk} = 1 \quad i = 1, 2, \ldots, n, j = 1, 2, \ldots, n_i. \tag{23}$$

Finally, the following mixed integer nonlinear programming problem would be approximately equivalent to the original model and from which $\lambda^* = \left[\lambda^*_{i_j}, i = 1, 2, \ldots, n, j = 1, 2, \ldots, n_i\right]^T$ or the optimal control vector is obtained.

$$
\begin{aligned}
& Min\, z \\
& s.t: \\
& \sum_{i=1}^{n} g_i\left(\sum_{j=1}^{n_i} \frac{1}{\lambda_{ij}}\right) - c_1 z \le b_1 \\
& \sum_{k=0}^{K} (1 - P_1(k))\Delta t + c_2 z \ge b_2 \\
& \sum_{k=0}^{K-1} (k\Delta t)^2 (P_1(k+1) - P_1(k)) - \left[\sum_{k=0}^{K-1} k\Delta t (P_1(k+1) - P_1(k))\right]^2 - c_3 z \le b_3 \\
& 1 - P_1\left(\left[\frac{u}{\Delta t}\right]\right) + c_4 z \ge b_4 \\
& P(k+1) = P(k) + Q(\lambda)P(k)\Delta t \quad k = 0, 1, \ldots, K-1 \\
& P_i(0) = 0 \quad i = 1, 2, \ldots, N-1 \\
& P_N(k) = 1 \quad k = 0, 1, \ldots, K \\
& P_i(k) \le 1 \quad i = 1, 2, \ldots, N-1,\ k = 1, 2, \ldots, K \\
& \lambda_{i_j} = \sum_{k=1}^{|S_{i_j}|} \alpha_{ijk} y_{ijk} \quad i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n_i \\
& \sum_{k=1}^{|S_{i_j}|} y_{ijk} = 1 \quad i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n_i \\
& y_{ijk} \in \{0, 1\} \quad i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n_i,\ k = 1, 2, \ldots, |S_{i_j}| \\
& z \ge 0
\end{aligned}
\tag{24}
$$

Table 1 Characteristics of the components of Case I

i	Distribution	Parameters	Purchase cost
1	Generalized Erlang	$(\lambda_{1_1}, \lambda_{1_2})$	$2\left(\frac{1}{\lambda_{1_1}}\right) + 3\left(\frac{1}{\lambda_{1_2}}\right) + 4$
2	Generalized Erlang	$(\lambda_{2_1}, \lambda_{2_2}, \lambda_{2_3})$	$\left(\frac{1}{\lambda_{2_1}}\right) + 5\left(\frac{1}{\lambda_{2_2}}\right) + 2\left(\frac{1}{\lambda_{2_3}}\right) + 3$
3	Exponential	λ_{3_1}	$6\left(\frac{1}{\lambda_{3_1}}\right)^2 + 5$
4	Exponential	λ_{4_1}	$3\left(\frac{1}{\lambda_{4_1}}\right)^2 + 7$

If we consider the initial and terminal state conditions for $P(k)$ implicitly and substitute each $P_i(0)$ and $P_N(k)$ with 0 and 1, respectively, the mixed integer nonlinear programming problem 24 would have $K(N-1)+1$ continuous decision variables and $\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left|S_{i_j}\right|$ $0-1$ decision variables.

For estimating the length of the time interval, we consider each λ_{i_j} as the median of S_{i_j} for i=1,2,...,n, j=1,2,..., n_i. Then, we solve the system of differential equations (Eq. 9) analytically to obtain $P_1(t)$, according to the values of λ_{i_j} taken from the previous step. A good estimation for the length of the time interval is given by $\widehat{T}$, in which $P_1(\widehat{T})$ should be greater than or equal to $1-\varepsilon$. We consider ε equal to 0.01 in this paper; consequently, $\widehat{T}$ can be computed numerically by solving the nonlinear equation $P_1(\widehat{T})$=0.99.

A computer program was written to evaluate our algorithm on some problems with different sizes and investigate the trade-off between the accuracy (correctness) and the computational time in each problem. At the beginning of the algorithm, we consider K=10 and $\Delta t = \widehat{T}\big/10$. In an accurate solution, $P_1(k)$ should approach 1. Otherwise, the value of Δt is increased to obtain a more accurate solution.

After solving the problem (Eq. 24) and obtaining λ^*, we compute $P_1(t)$ by analytically solving the system of differential equations with constant coefficients (Eq. 9). Then, we compute the exact MTTF and VTTF from Eqs. 11 and 12, respectively. The percentage difference between the approximated MTTF taken from Eq. 18 and the exact MTTF taken from Eq. 11 (PD.M) and also the percentage difference between the approximated VTTF taken from Eq. 19 and the exact VTTF taken from Eq. 12 (PD.V), or the absolute differences between the approximated values and the exact values divided by the exact ones, can be considered as two important criteria for the accuracy of the discrete time approximated solution. As K is increased and Δt is decreased, PD.M and PD.V approach 0. Therefore, the approximated discrete lifetime distribution approaches to the exact distribution because of matching the first two moments; consequently, the optimal solution of the discrete time problem (Eq. 24) approaches the goal attainment formulation of the original optimal control problem (Eq. 14).

In the next steps of the algorithm, we replace K with K+10 and each new value for Δt should be selected, such that the length of the time interval ($\widehat{T} = K\Delta t$) remains unchanged, to investigate the trade-off between optimality and computational time.

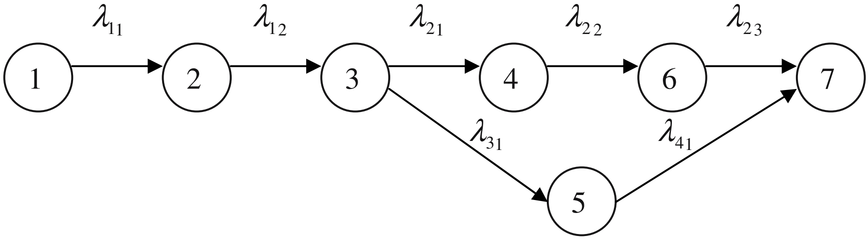

Fig. 4 Directed network of Case I

Table 2 Matrix $Q(\lambda)$ corresponding with Case I

State	1	2	3	4	5	6	7	8	9
1	$-\lambda_{1_1}$	λ_{1_1}	0	0	0	0	0	0	0
2	0	$-\lambda_{1_2}$	λ_{1_2}	0	0	0	0	0	0
3	0	0	$-(\lambda_{2_1} + \lambda_{3_1})$	λ_{2_1}	λ_{3_1}	0	0	0	0
4	0	0	0	$-(\lambda_{2_2} + \lambda_{3_1})$	0	λ_{3_1}	λ_{2_2}	0	0
5	0	0	0	0	$-(\lambda_{2_1} + \lambda_{4_1})$	λ_{2_1}	0	0	λ_{4_1}
6	0	0	0	0	0	$-(\lambda_{2_2} + \lambda_{4_1})$	0	λ_{2_2}	λ_{4_1}
7	0	0	0	0	0	0	$-(\lambda_{2_3} + \lambda_{3_1})$	λ_{3_1}	λ_{2_3}
8	0	0	0	0	0	0	0	$-(\lambda_{2_3} + \lambda_{4_1})$	$\lambda_{2_3} + \lambda_{4_1}$
9	0	0	0	0	0	0	0	0	0

The proposed methodology is easily generalized, in which not only the scale parameters (λ_{i_j}) but also the shape parameters (n_i) are considered as the design variables. In real-world problems, the designers sometimes use fundamentally different designs or technologies with different shape parameters because the failure mechanisms would be different. In this case, we first solve the optimization problem (Eq. 24) for all combinations of n_i for i=1,2,..., n. Then, the optimal n_i^*, i=1,2,..., n, would be related to that combination, which results the minimum z of the problem (Eq. 24). It should also be noted that the infinitesimal generator matrix for each combination of n_i would be different from the other combinations, and this matter clearly increases the complexity of the problem.

4 Computational experiments

For showing the numerical stability of the theoretical developments of the paper, we solve two numerical examples and investigate the trade-off between the accuracy and the computational time in each of them. In both examples, Saaty' method of pairwise comparisons (Hwang and Yoon 1981) can be used to compute the weights.

4.1 Case I

For space exploration, there are two space shuttles, which are depicted as in Fig. 1. In this system, there are two nonrepairable dissimilar units in a cold-standby redundancy scheme. At the beginning, the operating unit is unit 1, which is composed of shuttle A (component 1). When this shuttle fails, the redundant unit 2, which is composed of shuttle B (component 2), central controller I (component 3) and central controller II (component 4), as the cold-standby redundant components arranged in a series–parallel configuration, is put into operation. Table 1 shows the characteristics of the components.

The cost unit is in million dollars and the time unit is in year. The mission time, u, is assumed to be equal to 2 years. It is also assumed that $S_{i_j} = \{1, 1.1, 1.2, \ldots, 2\}$ for i=1,2,3,4, j=1,2,...,n_i. We set the goals for the initial purchase cost, MTTF, VTTF, and the system reliability at the mission time as b_1=30, b_2=2.7, b_3=0.5, and b_4=0.7, respectively. Since 1-year deviation from the system MTTF is known to be 20, 0.5, and 5 times as important as $1 million deviation from the initial purchase cost, 1-year deviation from the system VTTF and also one unit deviation from the system

Table 3 Trade-off results in Case I

No.	C	$MTTF_a$	$VTTF_a$	R_a	PD.M %	PD.V %	K	Δt	CT
1	37.368	2.499	0.164	0.766	0	87.98	10	0.5	52
2	36.101	2.394	0.653	0.677	2.44	48.3	20	0.25	5:42
3	35.839	2.296	0.702	0.561	1.54	37.88	30	0.167	16:11
4	35.5	2.227	0.733	0.537	1.68	30.91	40	0.125	18:38
5	35.206	2.184	0.749	0.522	1.75	26.49	50	0.1	22:49
6	31.134	1.977	0.78	0.443	1.73	9.19	500	0.01	48:34

Table 4 $\lambda^*_{i_j}$ for i=1,2,3,4, j=1,2,..., n_i, in Case I

$\lambda^*_{1_1}$	$\lambda^*_{1_2}$	$\lambda^*_{2_1}$	$\lambda^*_{2_2}$	$\lambda^*_{2_3}$	$\lambda^*_{3_1}$	$\lambda^*_{4_1}$
2	2	2	2	2	1.3	1.2

reliability, respectively, then c_1=0.7547, c_2=0.0377, c_3=0.0189, c_4=0.1887. The objective is to select the best components from the set of available components to be placed in this two dissimilar-unit multicomponent nonrepairable cold-standby redundant system.

First, we construct the proper directed network following Rule 1, as depicted in Fig. 4. The stochastic process $\{X(t), t \geq 0\}$, related to the shortest path analysis of this directed network, has nine states in the order of Ω^*={(1),(1,2),(1,2,3),(1,2,3,4),(1,2,3,5),(1,2,3,4,5),(1,2,3,4,6),(1,2,3,4,5,6),(1,2,3,4,5,6,7)}. Table 2 shows matrix $Q(\lambda)$.

The length of the time interval is approximated as $\widehat{T} = 5$. Therefore, we consider K=10 and Δt=0.5 at the beginning. Then, we formulate the proper multi-objective reliability optimization problem according to Eq. 24. For this problem, there are 77 0–1 decision variables. The number of prospective solutions to the problem is larger than 1.51^{23}. For investigating the trade-off between the accuracy and the computational time (CT mm:ss) on a PC Pentium IV 2.1-GHz processor, we also solve the problem for K=20,30,40,50,500 and compute the values of PD.M and PD.V in each case. Table 3 shows the results. In this table, all solutions are Pareto-optimal and satisfy the necessary condition ($P_1(K) \geq 0.99$).

Table 4 shows $\lambda^*_{i_j}$ for i=1,2,3,4, j=1,2,..., n_i, considering K=500 and Δt=0.01. In this case, PD.M and PD.V are almost equal to 1.7 and 9%, respectively. Therefore, the accuracy of this solution is acceptable, but access to this level of accuracy still needs a relatively long computational time (about 48 min).

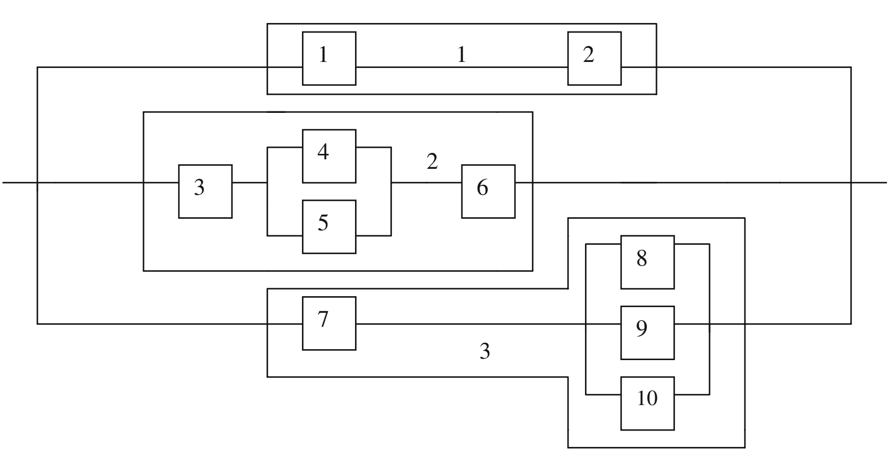

Fig. 5 Spacecraft controller of Case II

Table 5 Characteristics of the components of Case II

i	Distribution	Parameters	Purchase cost
1	Exponential	λ_{1_1}	$7\left(\frac{1}{\lambda_{1_1}}\right)^2+5$
2_1	Exponential	λ_{2_1} (n_2=1)	$2\left(\frac{1}{\lambda_{2_1}}\right)+2$
2_2	Generalized Erlang	$(\lambda_{2_1}, \lambda_{2_2})$ (n_2=2)	$\left(\frac{1}{\lambda_{2_1}}\right)+\left(\frac{1}{\lambda_{2_2}}\right)+2$
3	Exponential	λ_{3_1}	$8\left(\frac{1}{\lambda_{3_1}}\right)+68$
4	Exponential	λ_{4_1}	$4\left(\frac{1}{\lambda_{4_1}}\right)+3$
5	Exponential	λ_{5_1}	$4\left(\frac{1}{\lambda_{5_1}}\right)+3$
6	Exponential	λ_{6_1}	$10\left(\frac{1}{\lambda_{6_1}}\right)+4$
7	Exponential	λ_{7_1}	$3\left(\frac{1}{\lambda_{7_1}}\right)^2+7$
8	Exponential	λ_{8_1}	$5\left(\frac{1}{\lambda_{8_1}}\right)+2$
9	Exponential	λ_{9_1}	$5\left(\frac{1}{\lambda_{9_1}}\right)+2$
10	Exponential	λ_{10_1}	$5\left(\frac{1}{\lambda_{10_1}}\right)+2$

4.2 Case II

Case II, which is depicted in Fig. 5, shows the controller system of a spacecraft. In this system, there are three nonrepairable dissimilar units in a cold-standby redundancy scheme.

At the beginning, the operating unit is unit 1, which is composed of a laptop computer (component 1) and a power supply (component 2) arranged in a series configuration. When this unit fails, the redundant unit 2, which is composed of PC I (component 3), CD drive I (component 4), and CD drive II (component 5), as the cold-standby redundant components, and also a monitor (component 6) arranged in a series–parallel configuration, is put into operation. If unit 2 fails, then the redundant unit 3, which is composed of PC II (component 7) and hard drive I (component 8), hard drive II (component 9) and hard drive III (component 10), as the cold-standby redundant components, arranged in a series–parallel configuration, goes into operation.

Table 6 Trade-off results in Case II

No.	C	$MTTF_a$	$VTTF_a$	R_a	PD.M %	PD.V %	K	Δt	CT
1	111.714	2.937	0.71	0.849	0.71	71.62	10	0.6	7:39
2	112.656	2.59	0.943	0.724	0.77	46.36	20	0.3	9:23
3	110.937	2.46	1.053	0.677	1.16	34.31	30	0.2	10:56
4	107	2.347	1.072	0.637	1.18	27.76	40	0.15	20:34
5	107	2.341	1.13	0.636	1.43	23.85	50	0.12	22:41
6	89	2.043	1.125	0.532	1.25	7.02	500	0.012	56:38

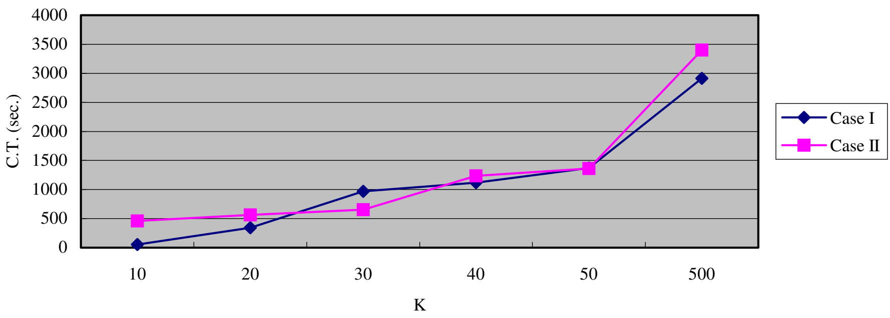

Fig. 6 Computational time (s) vs K

In all components, except component 2, the shape parameters (n_i) are considered equal to 1, because we suppose that the replacements have the same failure mechanisms. For component 2, it is supposed that there is also another replacement with generalized Erlang distribution lifetime of order n_2=2 and the parameters $(\lambda_{2_1}, \lambda_{2_2})$ except the first replacement with exponential lifetime. Table 5 shows the characteristics of the components.

The cost unit is in hundred dollars and the time unit is in year. The mission time u is assumed to be equal to 1.8. It is also assumed that $S_{i_j} = \{0.5, 0.6, \ldots, 1\}$ for i=1,2,...,10, j=1,2,...,n_i. We set the goals as b_1=100, b_2=3.5, b_3=0.5, and b_4=0.8. Under-attainment of the goals are assumed to be c_1=0.7547, c_2=0.0377, c_3=0.0189, c_4=0.1887, like the previous case.

The length of the time interval is approximated as $\widehat{T} = 6$. Therefore, we consider K=10 and Δt=0.6 at the beginning. Then, we formulate and solve the proper multi-objective reliability optimization problem for both combinations of n_2 (1 and 2) according to Eq. 24.

Table 6 shows the trade-off between the accuracy and the computational time for the different pairs of K and Δt. Considering K=500 and Δt=0.012, we obtain $\lambda^*_{i_j} = 1$ for all i=1,2,...,10 and j=1,2. Moreover, the optimal replacement for component 2 (power supply) would be a hardware with generalized Erlang distribution lifetime of order n^*_2=2 and the parameters $\left(\lambda^*_{2_1} = 1, \lambda^*_{2_2} = 1\right)$, and we should pay \$400 for purchasing this kind of hardware.

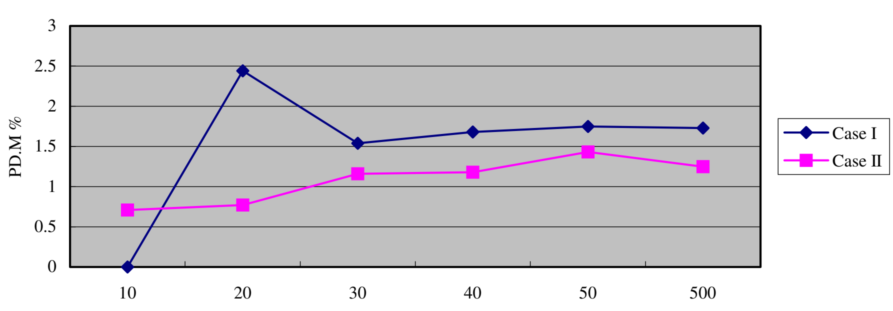

Fig. 7 PD.M vs K

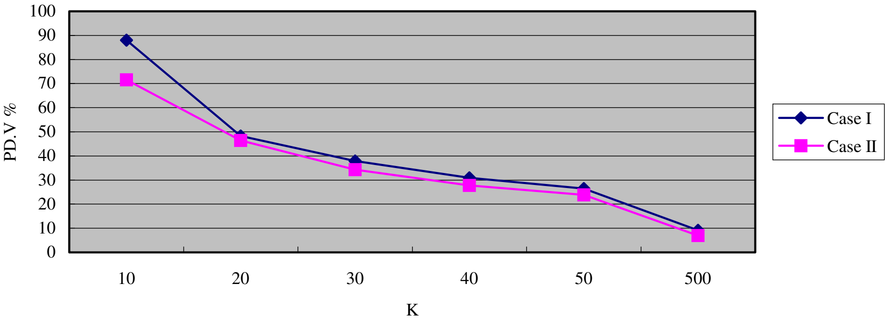

Fig. 8 PD.V vs *K*

Figures 6, 7 and 8 show the computational time, PD.M, and PD.V, respectively, according to the different pairs of K and Δt in two indicated cases. According to Fig. 6, computational time grows with K. Computational time is also strongly dependent on n_i, the network size, and the number of different shape parameters (n_i) for each component. According to Fig. 8, when K is increased, the percentage difference between the approximated and the exact variance of the system lifetime, which is one of the most important criteria for the accuracy of the solution, will be decreased.

5 Conclusion

In this paper, we introduced a new methodology for the reliability optimization of dissimilar-unit nonrepairable cold-standby redundant systems, in which each unit is composed of a number of independent components arranged in a series–parallel configuration. The system components work independently, and the lifetime of each component is a random variable with generalized Erlang distribution, in which the decision variables are both scale and shape parameters. We assumed that for each component, the distribution parameters are to be determined from among some discrete sets. The purchase cost of each component was also assumed to be an increasing function of its expected lifetime.

To select the desired components, we developed a goal attainment model with four conflicting objectives, minimization of the total purchase costs, maximization of the mean time to failure of the system, minimization of the variance of time to failure of the system, and also maximization of the system reliability at the given mission time. Then, in order to solve the resulting optimal control problem, it was transformed into a mixed integer nonlinear programming problem.

Although, at the first glance, it seems that the proposed mixed integer nonlinear programming problem has many continuous and 0–1 decision variables, but using the shortest path technique for reliability optimization has many other advantages over the classical approaches. Computing the reliability function of these standby systems using classical approaches, which is essential for reliability optimization, if not impossible, is at least so complicated for most real case problems because either the convolution integrals are intractable or the size of the state space would be enormous. For example, this problem could be solved by using clever complete enumeration of network states (Shooman 1991). According to our methodology, for a complete directed network with l nodes and $l(l-1)$ arcs representing the components of the system (the worst case example), the size of the state space would be $2^{l-2}+1$, but the size of the state space in Shooman's method would be equal to $3^{l(l-1)}$ because each component can be in one of these three states: work, fail, and standby. In the numerical example of Section 4.2, the size of the state space for $n_2=1$ is equal to 7 and for $n_2=2$ is equal to 8 (totally 15). However, according to the Shooman's method, the size of the state space would be 2×3^{10}, which is much larger than our proposed methodology.

According to the computational experiments, when K grows and Δt goes to 0, the percentage difference between the approximated and the exact mean would be about 0 and the percentage difference between the approximated and the exact variance approaches 0. Therefore, the approximated discrete lifetime distribution approaches to the continuous distribution, because the first two moments of the approximated distribution and the exact distribution are matched with each other. In this case, the optimal solution of the discrete time problem also approaches to the optimal solution of the original continuous time problem, accordingly. In more realistic-sized problems, the values of K and Δt should be selected such that we can solve the problem in an acceptable level of accuracy with reasonable computational time. According to these experiments, there is no significant relation between PD.M and the network size and also between PD.V and the network size, but the computational time grows with the size of the network, the value of n_i, and also the number of different shape parameters for each component.

The proposed model can be easily extended to the general types of nonconstant hazard functions. In the case of general distribution of lifetime, the lifetime distribution can be approximated by a generalized Erlang distribution, by matching the first three moments, because the generalized Erlang distributions are a special class of Coxian distributions and each general distribution can be easily approximated by a Coxian distribution.

References

Albright SC, Soni A (1984) Evaluation of costs of ordering policies in large machine repair problem. Nav Res Logist Q 31(3):387–398

Azaron A, Katagiri H, Sakawa M, Modarres M (2005a) Reliability function of a class of time-dependent systems with standby redundancy. Eur J Oper Res 164(2):378–386

Azaron A, Katagiri H, Kato K, Sakawa M (2005b) Reliability evaluation and optimization of dissimilar-component cold-standby redundant systems. J Oper Res Soc Jpn 48(1):71–88

Birolini A (1994) Quality and reliability of technical systems. Springer, Berlin, Heidelberg, New York

Bulfin RL, Liu CY (1985) Optimal allocation of redundant components for large systems. IEEE Trans Reliab R-34:241–247

Coit DW (2001) Cold-standby redundancy optimization for nonrepairable systems. IIE Trans 33 (6):471–478

Dhingra A (1992) Optimal apportionment of reliability & redundancy in series systems under multiple objectives. IEEE Trans Reliab 41(4):576–582

Fyffe DE, Hines WW, Lee NK (1968) System reliability allocation and a computational algorithm. IEEE Trans Reliab 17:64–69

Gen M, Ida K, Lee JU (1990) A computational algorithm for solving 0–1 goal programming with GUB structures and its application for optimization problems in system reliability. Electron Commun Jpn Part 3, 73:88–96

Gnedenko B, Ushakov I (1995) Probabilistic reliability engineering. Wiley, New York

Goel LR, Gupta R (1983) Reliability analysis of multi-unit cold standby system with two operating modes. Microelectron Reliab 23:1045–1050
Goel LR, Gupta R, Gupta P (1983) A single unit multi-component system subject to various types of failures. Microelectron Reliab 23(5):813–816
Gupta R, Bajaj CP, Sinha SM (1986) A single server multi-component two-unit cold standby system with inspection and imperfect switching device. Microelectron Reliab 26:873–877
Gupta R, Mumtaz SZ, Goel R (1997) A two dissimilar unit multi-component system with correlated failures and repairs. Microelectron Reliab 37:845–849
Hwang CL, Yoon K (1981) Multiple attribute decision making, Springer, Berlin, Heidelberg, New York
Kulkarni VG (1986) Shortest paths in networks with exponentially distributed arc lengths. Networks 16(3):255–274
Kumar S, Kumar D, Mehta NP (1996) Behavioral analysis of shell gasification and carbon recovery process in a urea fertilizer plant. Microelectron Reliab 36(5):671–673
Nakagawa Y, Miyazaki S (1981) Surrogate constraints algorithm for reliability optimization problems with two constraints. IEEE Trans Reliab 30:175–180
Nakashima K, Yamato K (1977) Optimal design of a series–parallel system with time-dependent reliability. IEEE Trans Reliab 26(2):119–120
Pandey D, Jacob M, Yadav J (1996) Reliability analysis of a powerloom plant with cold-standby for its strategic unit. Microelectron Reliab 36(1):115–119
Prasad VR, Kuo W, Oh-Kim KM (1999) Optimal allocation of s-identical, multi-functional spares in a series system. IEEE Trans Reliab 48(2):118–126
Robinson DG, Neuts MF (1989) Standby redundancy in reliability: a review. IEEE Trans Reliab 38(4):430–435
Sakawa M (1978) Multiobjective optimization by the surrogate worth trade-off method. IEEE Trans Reliab 27(5):311–314
Sakawa M (1980) Reliability design of a standby system by a large-scale multiobjective optimization method. Microelectron Reliab 20(3):191–204
Sethi S, Thompson G (1981) Optimal control theory. Martinus Nijhoff Publishing, Boston
Shooman M (1991) Probabilistic reliability: an engineering approach, 2nd edn. Krieger, Melbourne
Sinaki G (1994) Ultra-reliable fault tolerant inertial reference unit for spacecraft. In: Proceedings of the Annual Rocky Mountain Guidance and Control Conference, Univelt Inc., San Diego, CA
Srinivasan SK, Subramanian R (1980) Probabilistic analysis of redundant systems. Lecture notes in economics and mathematical systems, no. 175. Springer, Berlin, Heidelberg, New York
Yamashiro M (1981) A repairable system with partial and catastrophic failure modes. Microelectron Reliab 21(1):97–101